The Hometown Discount Paradox

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Abstract

A player is said to offer a team a "hometown discount" if he prefers signing with that team for less money than a competitor. At first thought, hometown discounts appear to only assist teams in signing players. However, we show that hometown discounts can actually hinder agreements under a reasonable set of assumptions. To prove this, we develop a model in which the team is uncertain of how much of a hometown discount the player is willing to give. In equilibrium, the hometown team fails to sign the player with positive probability even though it would always sign the player if it was common knowledge that the player was not willing to offer a hometown discount. As such, it is not clear whether hometown discounts assist a team in signing players, even though the possibility of such a discount increases the team's payoff in expectation.

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1 Introduction

A player is said to offer a team a "hometown discount" if he prefers signing with that team for less money than a competitor offers him. Teams rightfully seek out players willing to give a hometown discount, as the team can come to terms with the player at a below-market rate. For that reason, it appears hometown discounts ought to facilitate signings and unconditionally simplify the negotiating process.

This research note qualifies that intuition. Although hometown discounts indeed (weakly) increase a team's payoff during contract negotiations, it can also *prevent* agreement because of the uncertainty that comes along with it. While a team might know that a player will give some sort of discount, it seems reasonable to believe that the maximum discount he will sacrifice is uncertain from the team's perspective. When the player receives competitive offers from rival teams–thus driving up the equilibrium price of the contract regardless of the discount–the team optimally gambles with its contract terms, offering an amount which only players giving large hometown discounts are willing to accept. Consequently, the team fails to sign some players even though Pareto improving agreements exist with complete information.

This note proceeds as follows. We begin by considering a complete information to formally introduce the notion of a hometown discount, which allow teams to sign a subset of players with good outside options that it would otherwise be unable to obtain. We then allow the size of the hometown discount to be private information to the player. When player's outside options are competitive, bargaining occasionally breaks down due to the existence of the discount. Finally, the conclusion briefly discusses the empirical implications of the findings.

2 The Model

Consider a simple model of the free agency process. There are two moves. First, the team offers the player a contract worth $x \geq 0$ dollars. The player observes the offer amount and chooses to accept or reject it. If the player accepts, the contract becomes official. If the player rejects, we assume he signs the most competitive contract offer elsewhere, which is exogenous to the game at hand.

As Figure 1 illustrates, payoffs are as follows. The team values the player worth $V > 0$. If the player signs with the team, the team receives the benefit V but pays the cost of the contract x, for an overall payoff of $V - x$. If the player signs with the team, he receives the contract size x and an additional benefit of playing with his hometown team $h > 0$ for a payoff of $x + h$. Note that higher levels of h indicate the player is willing to sacrifice more to play with that particular team. If the player rejects, he signs elsewhere for $n \geq 0$ dollars, for a payoff of n. The value of this contract from a rival team is commonly known to both players throughout.

With the preliminaries out of the way, we can state our first result:

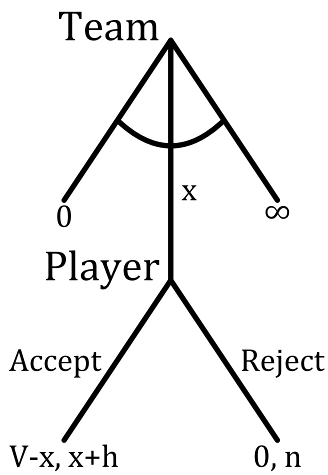

Figure 1: The extensive form of the complete information game.

Proposition 1: For sufficiently large hometown discounts, the team always signs the player.

Proposition 1 formalizes the conventional wisdom regarding hometown discounts. Proving it is straightforward. Since the interaction is sequential and of complete information, backward induction suffices as a solution concept. When the player moves, he is willing to sign with the team if the contract offer plus the hometown discount is worth more than the offer from elsewhere, or:

$$x + h \geq n$$

$$x \geq n - h$$

Recall that the team's payoff is $V - x$, which is strictly decreasing in x. As such, its optimal acceptable offer is $x = n - h$ if $n > h$ or $x = 0$ if $n < h$.[1] In turn, the team can choose between two types of offers: the optimal offer $x = max\{n - h, 0\}$ or an offer $x < n - h$ if such an offer exists.

First, consider the trivial case when $n < h$. Here, no matter the team's offer, the player signs. Since the team's payoff is strictly decreasing in x, the team simply offers $x = 0$, the player signs, and the game ends.

Second, suppose $n > h$. This time, the player can offer $x < n-h$ and induce the player to reject. The earns 0 for this outcome. In contrast, it can offer $x = n - h$ and induce the player to accept. The latter is optimal if:

$$V - n + h > 0$$

$$V > n - h$$

[1]The case where $n = h$ is irrelevant, as it is a knife-edge condition.

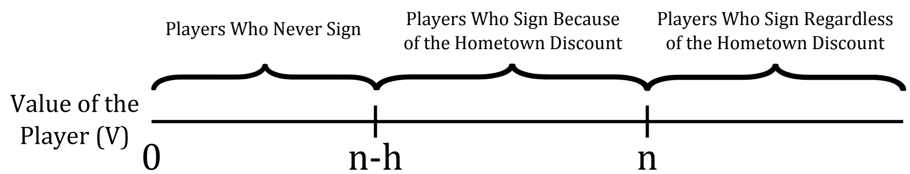

Figure 2: The outcome of the game as a function of the value of the player to the team. The hometown discount is necessary for the team to sign players with values between $n - h$ and n.

Therefore, the team signs the player if $V > n - h$, while the player signs with a rival if $V < n - h$.[2] As such, provided the player's hometown discount is large enough, the player signs with the team.

Before moving on, there are a couple of remarks. With complete information, the team's payoff is only weakly increasing in the hometown discount. If the team does not value the player much, even large hometown discounts are not sufficient to make the team want to sign the player; although the team can sign the player for less money, it is still not willing to match the competing offer even after factoring in the discount.

However, as Figure 2 shows, the hometown discount allows teams to sign players it otherwise could not. If no hometown discount exists ($h = 0$), the team only signs the player if $V > n$. With the hometown discount, the team signs the player if $V > n - h$. As such, the hometown discount only affects the signing outcome of players with values $V \in (n - h, n)$. Without discount, the players do not sign with the team; but with the discount, they do. This subset of players reflects the traditional notion of the hometown discount advantage.

2.1 When the Discount Is Private Information

While a player may clearly have a hometown preferences for a particular team, the extent of the discount is better modeled as private information to the player. After all, the player's fondness for his home is intrinsic and not directly observable. There may be indications–place of birth, location of family, ownership of a house–but it is not apparent exactly how much a player is willing to sacrifice to maintain these benefits. Moreover, signalling is not possible here, as the player has strategic incentive to underrepresent his hometown value and force the team to offer him a larger contract.[3]

To investigate how this private information affects bargaining, suppose nature begins the game by drawing the player's hometown discount h from a

[2] The game has multiple equilibria when $V < n - h$, as the team can optimally offer *any* amount $x < n - h$. However, all of these equilibria are payoff and outcome equivalent–the player always signs with the rival.

[3] Even after the player has signed the contract, the player has incentive to misrepresent his preferences over the location. This time, however, he prefers *overrepresenting* the city's desirability to curry favor with the fanbase. Thus, we cannot take players at their word at any point.

commonly known distribution uniform distribution in the interval $(0, \bar{h})$, where $\bar{h} < n - x$.[4] The player observes his own level of h, but the team does not. Therefore, when the team makes its offer, it cannot update its beliefs about h, but the player conditions whether to accept or reject based off of the drawn value of the discount.

To stack the deck against the hometown discount affecting whether the player signs with the team in equilibrium, suppose $V > n$. Thus, if the team knew the player's realized hometown discount, it could offer the player $n - h$ and sign the player, as in the complete information model.

Since this a sequential game of incomplete information, perfect Bayesian equilibrium is the appropriate solution concept. A perfect Bayesian equilibrium is a set of strategies and beliefs such that the strategies are rational given the beliefs and the beliefs are updated via Bayes' rule wherever possible.[5]

We can now state the result:

> **Proposition 2**: For sufficiently competitive outside offers, the team offers contracts which the player rejects with positive probability.

Perfect Bayesian equilibrium is straightforward for this game. As previously mentioned, since the informed actor (the player) only moves at the end of the interaction, the team's posterior belief when it makes the offer is simply its prior. Consequently, we only need to solve for the optimal strategies of each player.

First, consider the player's choice. The player is fully informed and sees the team's offer size x. Thus, the player accepts if $x+h$ is greater than n and rejects if n is greater than $x + h$.[6]

Given that, the team optimizes its contract offer according to its prior belief whether the player will accept or reject it, weighed according to the team's payoff for each of these outcomes:

$$Eu(x) = Pr[accept](V - x) + Pr[reject](0)$$
$$Eu(x) = Pr[accept](V - x)$$

Recall that h is uniformly distributed on the interval $(0, \bar{h})$. Therefore, we may write the probability the player accepts an offer x as:

$$Pr[accept] = Pr[x > n + h]$$
$$Pr[accept] = Pr[h < n - x]$$
$$Pr[accept] = 1 - \frac{n - x}{\bar{h}}$$

[4] This final restriction ensures that no type would be willing to sign for contracts worth nothing.

[5] See Fudenberg and Tirole 1991.

[6] What occurs when $x + h = n$ is inconsequential, as such an individual exists with probability mass 0.

This is a valid probability for $x \in [0, n]$. But note that $x > n$ cannot be optimal for the team, as offering $x = n$ also induces all types of players to accept but for a lower overall amount. Knowing this, we can solve for the first order condition of the team's optimal offer:

$$Eu(x) = \left(1 - \frac{n - x}{\bar{h}}\right)(V - x)$$

$$Eu'(x) = \frac{V + n - 2x}{\bar{h}} - 1 = 0$$

$$x^* = \frac{V + n - \bar{h}}{2}$$

Since $V > n$ implies that inducing all types to reject is not optimal for the team, there are two critical points to check: $x = n$ and $x = x^*$. If $x^* > n$, clearly $x = n$ is the maximum; offering x^* instead still induces all types to sign but includes an unnecessary concessions. Thus, the interesting case is when $x^* < n$. In this case, if the team offers n, all types accept, and the team earns $V - n$. Alternatively, the team could offer $x^* = \frac{V+n-\bar{h}}{2}$, which induces all types with $h > \frac{\bar{h}+n-V}{2}$ to accept while all other types reject. The team earns $(1 - \frac{n-x^*}{\bar{h}})(V - x^*)$ for this outcome. Thus, x^* is the superior offer if:

$$\left(1 - \frac{n - x^*}{\bar{h}}\right)(V - x^*) > V - n$$

A large amount of simplification eventually yields:

$$[\bar{h} - (V - n)]^2 > 0$$

To see this holds, note that $x^* < n$ implies $\bar{h} > V - n$. A square of a strictly positive number is strictly positive, so the inequality holds. Therefore, the team optimally offers x^* if $x^* < n$ and $x = n$ otherwise.

Note that when $x^* < n$, the player rejects with positive probability. We can calculate that exact probability using the formula for the probability of acceptance from before:

$$Pr[accept] = 1 - \frac{n - x^*}{\bar{h}}$$

$$Pr[accept] = 1 - \frac{n - \frac{V+n-\bar{h}}{2}}{\bar{h}}$$

$$Pr[accept] = \frac{1}{2} + \frac{V - n}{2\bar{h}}$$

Since $V > n$, more than half of the types accept. However, a sizeable share of the players nevertheless reject, particularly if the team faces a competitive opposing offer. Figure 3 illustrates this dynamic, plotting the probability the player accepts the team's contract as a function of the outside option n. Intuitively, if n is close to V, the team does not benefit much from signing the

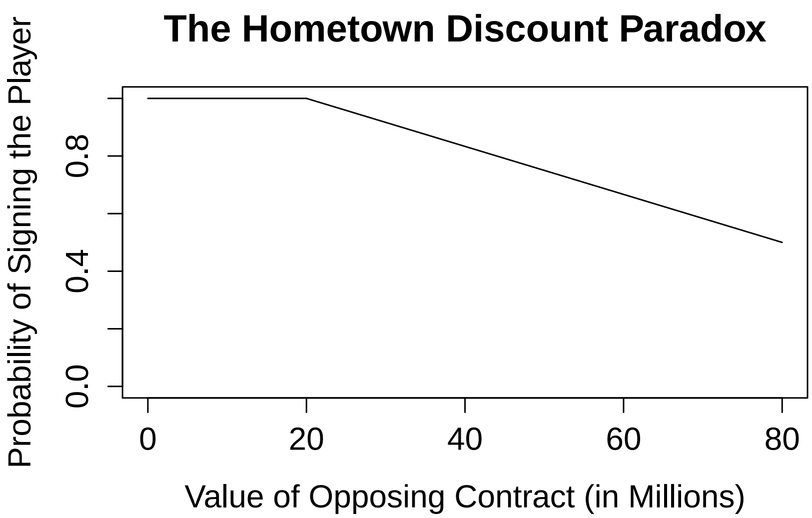

Figure 3: The probability the team signs the player as a function of the player's outside option n. The other parameters are set to $V = 80$ million and $\bar{h} = 60$. When the outside option is poor, the team can offer the player such a small amount that it is not worth gambling on an even smaller contract. However, when the outside offer is competitive, the team risks not signing the player to receive a greater payoff when the player does accept.

player. Thus, it is willing to gamble with low contract offers and induce only the types donating great hometown discounts to accept, since those types give the team the greatest payoff. In contrast, when the difference between V and n is large, the team earns so much from signing the player that it does not risk alienating any of the types; the risk of not receiving the great benefit of the player at such a low cost is not worth shaving additional value by factoring in a hometown discount into the contract.

To reiterate, the team weakly benefits from the knowledge that the player is willing to take some sort of hometown discount. Figure 4 illustrates the team's gain. From $n = 0$ to $V - \bar{h}$, the team offers the same contract value it would if there were no hometown discount, so its payoff remains the same. However, at $n = V - \bar{h}$, the team begins earning more from using its knowledge of the possibility of a hometown discount to make the optimal gamble. Whereas the team's payoff converges to 0 as n approaches V if the team offers a contract guaranteed to sign everyone, its payoff for the gamble remains strictly positive. Thus, it is correct to say that the team's payoff increases with the hometown discount, but whether it increases the likelihood the player signs with the team depends on the nature of the discount.

From the player's perspective, the gamble can prove beneficial or ruinous.

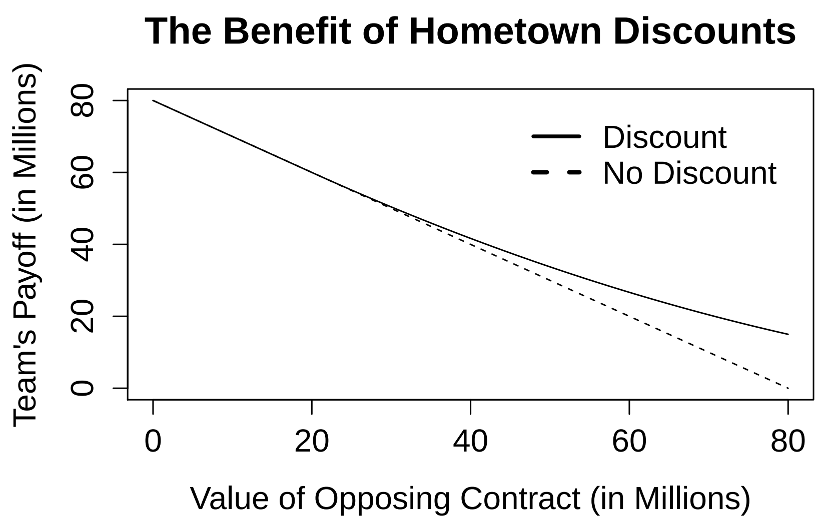

Figure 4: The team's payoff as a function of the outside option for when a private hometown discount exists and for when it is common knowledge that it does not exist. The parameters remain the same as in Figure 3. When the outside option is poor, the team offers the player a contract value as if no hometown discount existed, which means these two informational structures are payoff equivalent. However, when the outside option is good, the team risks not signing the player to obtain a better deal when there is incomplete information, which improves the team's payoff versus the complete information game.

Since the team makes an offer a subset of players accepts, those players willing to pay larger hometown discounts incur a smaller sacrifice than they are willing to make. However, players with small hometown discounts are shut out and sign for n with the competitor. Yet, if the player could credibly reveal its type to the team, any contract valued at $x \in [n - h, n]$ is Pareto improving for the team and player.

3 Conclusion

This note challenged the notion that hometown discounts always encourage a team to sign a player. Although that holds in the complete information scenario, uncertainty over the size of the hometown discount leads the team to gamble on smaller contracts in some cases. While the team earns a greater payoff than had it chosen an amount that all types would accept, the risky offer produces a greater payoff in expectation.

If a major purpose of formal theory is to create better-informed empirical studies, this note has shown that hometown discounts matter. Moreover, the extent of a hometown discount can vary from individual to individual. When constructing empirical models of contract values, restricting hometown discounts to a single variable is insufficient. Instead, we ought to consider including various factors related to hometown discounts–location, age, marital status, number of children–as independent variables.

4 References

Fudenberg, Drew and Jean Tirole. 1991. *Game Theory*. Cambridge: MIT Press.