Some Game Theory and Baseball

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Three Topics

1. How does a runner on third base affect pitching and batting strategies?
2. When is bunting for a base hit optimal?
3. How can you catch a baseball at a game?
Methodology: Game Theory

- Game theory is the study of strategic interaction.
- An environment is strategic if how I behave affects your welfare and how you behave affects my welfare.
Methodology: Game Theory

• Game theory is the study of strategic interaction.

• An environment is strategic if how I behave affects your welfare and how you behave affects my welfare.

  – Example: The type of pitch you throw affects my ability to hit, and the type of pitch I anticipate affects your ability to stop a hit.
Methodology: Game Theory

- Game theory is the study of strategic interaction.
- An environment is strategic if how I behave affects your welfare and how you behave affects my welfare.
- Game theory gives us tools to find how players optimally act and react to each other.
Who Am I?

- William SPANIEL
- PhD student in political science.
- gametheory101.com
- YouTube: Game Theory 101
- Game Theory 101: The Textbook
Who Am I?

• Bottom line: I am an evangelical game theorist.
• My goal in life is to make game theory clear to people who don’t spend their entire lives studying it.
Who Am I?

• Bottom line: I am an evangelical game theorist.
• My goal in life is to make game theory clear to people who don’t spend their entire lives studying it.
• Today, I will be horribly unclear. Sorry.
Three Topics

1. How does a runner on third base affect pitching and batting strategies?

2. When is bunting for a base hit optimal?

3. How can you catch a baseball at a game?
A Simple World

- Bases empty.
- Pitcher can throw two pitches: fastball or slider.
- Batter can anticipate fastball or slider.
- Batter wants to guess correctly; pitcher wants to make the batter guess incorrectly.
### With the Bases Empty

<table>
<thead>
<tr>
<th></th>
<th>Fastball</th>
<th>Slider</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batter</td>
<td>1, -1</td>
<td>0, 0</td>
</tr>
<tr>
<td>Slider</td>
<td>0, 0</td>
<td>1, -1</td>
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</table>
Fact: Being predictable is a bad strategy.

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<td>0, 0</td>
</tr>
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<td></td>
<td>0, 0</td>
<td>1, -1</td>
</tr>
</tbody>
</table>

- Batter fastball, Pitcher fastball: 1 point for Batter, -1 point for Pitcher.
- Batter fastball, Pitcher slider: 0 points for both.
- Batter slider, Pitcher fastball: 0 points for both.
- Batter slider, Pitcher slider: 1 point for Batter, -1 point for Pitcher.
Fact: Randomizing is an unbeatable strategy.

<table>
<thead>
<tr>
<th></th>
<th>Fastball (1/2)</th>
<th>Slider (1/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Batter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slider (1/2)</td>
<td>1, -1</td>
<td>0, 0</td>
</tr>
<tr>
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With a Runner on Third

• But what if a runner is on third base?
• Throwing a slider becomes risky.
  – If the ball goes past the catcher, the runner on third will score even though the player did not record a hit.
• The pitcher should throw sliders less frequently.
With a Runner on Third

• But what if a runner is on third base?
• Throwing a slider becomes risky.
  – If the ball goes past the catcher, the runner on third will score even though the player did not record a hit.
• The pitcher should throw sliders less frequently. Right?
Breaking Balls with a Runner on Third

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<tbody>
<tr>
<td>Batter</td>
<td>1, -1</td>
<td>X, -X</td>
</tr>
<tr>
<td></td>
<td>0, 0</td>
<td>1 + X, -1 - X</td>
</tr>
</tbody>
</table>

Pitcher
### Batter’s Payoff for Guessing FB

Let $p = \text{Pr(Throw FB)}$  

- Hitter earns $1$ with probability $p$.  
- Hitter earns $X$ with probability $1 - p$.  
- The payoff is given by:  
  $$1 \cdot p + (1 - p) \cdot X$$

<table>
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<th>Slider</th>
</tr>
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<td><strong>Fastball</strong></td>
<td>$1, -1$</td>
<td>$X, -X$</td>
</tr>
<tr>
<td><strong>Slider</strong></td>
<td>$0, 0$</td>
<td>$1 + X, -1 - X$</td>
</tr>
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</table>
Batter’s Payoff for Guessing Slider

- Let $p = \Pr(\text{Throw FB})$
- Hitter earns 0 with probability $p$.
- Hitter earns $1 + X$ with probability $1 - p$
- $(p)(0) + (1 - p)(1 + X)$

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Optimal Pitching Strategy

• The pitcher’s strategy is “unbeatable” if:
  • \((1)(p) + (1 - p)(X) = (1 - p)(1 + X)\)
  • \(p + X - pX = 1 + X - p - pX\)
  • \(2p = 1\)
  • \(p = \frac{1}{2}\)
    • That is, when the pitcher throws a fastball just as frequently as he throws a slider!
What the Frak?

• Why does the pitcher still throw sliders even though they are risky?
  – Suppose he threw fastballs more frequently then before.
  – The batter can respond by zeroing in on fastballs. Even though wild pitches aren’t a problem, the pitcher gets hammered anyway.
  – The pitcher therefore maintains his same randomizing strategy with a runner on third.
Pitcher’s Payoff for Throwing FB

- Let \( q = \text{Pr(Guess FB)} \)
- Pitcher earns -1 with probability \( q \).
- Pitcher earns 0 with probability \( 1 - q \).
- \((-1)(q) + (1 - q)(0)\)

\[
\begin{array}{c|c|c}
\text{Slider} & \text{Fastball} & \text{Slider} \\
\hline
\text{Fastball} & 1, -1 & X, -X \\
\hline
0, 0 & 1 + X, -1 - X & \\
\end{array}
\]
Pitcher’s Payoff for Throwing Slider

- Let $q = \Pr(\text{Guess FB})$
- Pitcher earns $-X$ with probability $q$.
- Pitcher earns $-1 - X$ with probability $1 - q$
- $(-X)(q) + (1 - q)(-1 - X)$
Optimal Batting Strategy

• The batter’s strategy is “unbeatable” if:
  • 
  • (-1)(q) + (1 – q)(0) = (-X)(q) + (1 – q)(-1 – X)
  • -q = -qX – 1 – X + q + qX
  • 2q = 1 + X
  • q = (1 + X)/2

• The batter guesses fastball more frequently!
  – He plays it safe—even if he guesses fastball incorrectly, the runner will sometimes score anyway.
Three Topics

1. How does a runner on third base affect pitching and batting strategies?
2. When is bunting for a base hit optimal?
   - “To Bunt or not to Bunt: Optimal Batting Strategy during a No-Hitter” (Working paper; Google it.)
3. How can you catch a baseball at a game?
Optimal Defensive Positioning

• Infield positioning affects optimal hitting strategies.
  – If the infielders are really close, bunting is bad.
  – If the infielders are really far away, bunting is good.

• What is the optimal defensive positioning? What is the optimal hitting strategy in response to that positioning?
The Interaction

• A simple two move game:
  1. Infielders pick a position.
  2. Batter sees their positioning and chooses whether to swing or bunt.

• For simplicity, consider just the third baseman’s strategy.
Some Logic

• Suppose the batter wants to bunt if the fielder is at 1 and wants to swing if the batter is at 0.
Some Logic

• If the batter bunts, his win percentage is increasing as the fielder moves closer to 1.
• If the batter swings, his win percentage is increasing as the fielder moves closer to 0.
Some Logic

- Intermediate value theorem: there is a unique fielding position at which the batter is indifferent between bunting and hitting.
Some Logic

- This is the optimal fielding position.
  - If the fielder moves closer, the hitter swings, and the fielder loses more frequently.
  - If the fielder moves further away, the hitter bunts, and the fielder loses more frequently.
Interesting Facts

• The fielder has a uniquely optimal fielding position.
  – Any other position is bad strategy.
• When the fielder plays this position, *it doesn’t matter* whether the hitter bunts or swings.
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Catching Baseballs Is Easy If...

- you are a nubile girl.
- you are under the age of 7.
- a player accidentally soaks you with beer.
- you know how to say “could you give me a ball please?” in obscure languages that some random baseball player speaks.
Catching Baseballs Is Easy If...

- you are a nubile girl.
- you are under the age of 7.
- a player accidentally soaks you with beer.
- you know how to say “could you give me a ball please?” in obscure languages that some random baseball player speaks.
- you know what you are doing.
Tip #1

- Go where they aren’t.
Tip #1

- Survey the possible areas that balls could go and how frequently they will go there.
- Note where people are congregating.
- Go to the place where the most balls go divided by the number of people there.
Tip #2

- Box out your neighbor.
Upper Deck
Upper Deck
Upper Deck
Upper Deck
Upper Deck

Upper Deck
Thanks!

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