Chapter 3

The Theory of Butter-for-Bombs Agreements

In this chapter, we develop a formal model of costly power shifts that shows that butter-for-bombs settlements can be sustainable in the long term, even if the rising state can freely renege. Depending on the parameters, the interaction ends in one of three ways. First, if the extent of the power shift is too great, the declining state can credibly threaten preventive war, which in turn makes the rising state's threat to build weapons incredible. Likewise, if the rising state's cost of building is too high, the declining state knows the rising state will never build. Either way, the declining state can offer the rising state no concessions and still induce acceptance. The outcome mirrors a world in which the rising state had no ability to shift power.

Second, if the threat to build is credible but investment costs remain relatively large, the declining state optimally offers immediate concessions to the rising state. The rising state accepts those concessions in the present and continuously in the future. Although the rising state could build and force the declining state to give yet more concessions, those additional concessions do not cover the cost of building. Thus, the rising state extracts concessions using *unrealized* power and maintains the status quo because of the attractiveness of future offers. This in turn allays the fears of the declining state.

Finally, if the cost of shifting power is low, the declining state cannot cheaply buy off the rising state. As a result, the declining state chooses to shortchange the rising state initially, forcing the rising state to shift power. Afterward, the declining state makes great concessions. The declining state could still induce the rising state not to build here, but it simply profits more from stealing as much as it can upfront. Put differently, the declining state's opportunism–not the rising state's opportunism–leads to the shift in power.

The results of the model indicate that, in the context of a bargaining game, the demand for proliferation is rare. In some cases, the declining state's threat of preventive war deters the rising state from building. In other cases, the rising state finds weapons more costly than useful. In between, the declining state can buy off some of the remaining states. Proliferation only occurs in the model when the investment cost is low.[1]

This chapter has three additional sections. The next section formally defines the model, describes some key features of the interaction, and derives its solution; in equilibrium, declining states and rising states reach peaceful, stable agreements if the cost to shift power falls within a certain range. After, we interpret the results. We then broadly illustrate the implications of the model on arms investment, negotiated agreements, and preventive war. A brief conclusion follows.

3.1 Modeling Butter-for-Bombs Agreements

This section introduces the central bargaining model of the book. We begin by describing the strategic interaction. Next, we highlight the key features of the model that depart from previous formal work on shifting power. With that, we then derive the game's equilibria and show that the declining state sometimes offers immediate concessions to convince the rising state not to build, even when conditions appear ripe for proliferation. We then interpret the results and show that they are robust to a number of alternative specifications.

3.1.1 Actions and Transitions

Consider an infinite period game between two actors, D (the declining state) and R (the rising state), as illustrated in Figure 3.1.[2] The states bargain

[1] However, as we will see in the next chapter, states have incentive to create equilibrium institutions to artificially raise the cost of building. Thus, an inefficiency puzzle remains, which the remainder of the book will address.

[2] We use these labels as a convention from the literature. In the basic model, the rising state rarely rises and the declining state rarely declines. Proliferation decisions occur more

over a good standardized to value 1. There are four states of the world: the pre-shift bargaining state, the post-shift bargaining state, the pre-shift war state, and the post-shift war state.

The game begins in the first period in the pre-shift bargaining state. In that state, D makes a temporary offer $x_t \in [0,1]$ to R, where t denotes the period. R accepts, rejects, or builds in response. If R rejects, the game transitions into the pre-shift war state, which is absorbing. R receives $p_R \in [0,1)$ in expectation while D receives $1-p_R$. These payoffs persist through all future periods, but the states pay respective costs $c_D, c_R > 0$ in each future period regardless.[3]

If R accepts, the period ends. R receives x_t for the period while D receives $1 - x_t$. The game then returns to the pre-shift bargaining stage, where D makes another temporary offer x_{t+1}.

If R builds, it pays a cost $k > 0$ to begin constructing the new weapons.[4] D sees this and decides whether to initiate a preventive war or advance to the post-shift state of the world.[5] Preventive war ends the game in the same pre-shift war state, as though R had rejected D's offer x_t. If D advances, the period ends, R receives x_t for the period while D receives $1 - x_t$, and the game transitions into the post-shift bargaining state. Similar to before, D makes an offer y_{t+1} to R. If R accepts, the period ends, R receives y_{t+1} for the period, D receives $1-y_{t+1}$ for the period, and the game repeats the post-shift bargaining state, where D makes another offer y_{t+2}. If R rejects, the game transitions into the post-shift war state, which is also an absorbing state. Here, R takes $p'_R \in (p_R, 1]$ in expectation while D receives $1 - p'_R$. These payoffs again persist through time, but the sides still pay their respective costs c_D, c_R.[6]

frequently in the extensions explored in later chapters.

[3]The results are the same if costs are only paid in the period of fighting. Moreover, the proof is identical except that we must substitute c_i with c'_i, where $c'_i = \frac{c_i}{1-\delta}$.

[4]Since we standardize the bargaining good as worth 1, k implicitly reflects R's resolve as well.

[5]In Chapter 8, we relax this assumption so that D has no direct knowledge whether R built.

[6]Note that the role of nuclear weapons is mundane. Successful proliferation merely increases the attractiveness of R's outside option while decreasing the attractiveness of D's outside option. This is consistent with Beardsley and Asal's (2009) empirical analysis of militarized interstate disputes that include a nuclear state. Kroenig (2013) similarly finds that larger nuclear arsenals increase the likelihood of prevailing in nuclear crises, though Sechser and Fuhrmann (2013) find evidence that nuclear weapons can only make

The states share a common discount factor $\delta \in (0,1)$. Thus, the states discount period t's share of the good and costs paid by δ^{t-1}.

3.1.2 Key Features

Before solving for the game's equilibria, we should highlight four important features of the model and how they differ from previous attempts to understand bargaining and power shifts. First, following the second wave of shifting power research (Jackson and Morelli 2009; Chadefaux 2011; Fearon 2011; Debs and Monteiro 2013), the power shift is costly and endogenous. These are minimalist and necessary criteria. The vast majority of major power shifts result from endogenous choices made by rising states (Debs and Monteiro 2013, 4-5). Moreover, keeping power shifts exogenous prohibits the states from bargaining over weapons, since strength appears by assumption. And if weapons were not costly, states would have no incentive to bargain over them, as arms building would be an efficient process.

Second, we allow the interaction to continue forever. If the rising state were to lose the ability to proliferate at any point, it would have to build in the periods previous to force the declining state to offer concessions. As such, the rising state maintains the ability to proliferate in every pre-shift period. Future chapters will later address what happens if the rising state might be unable to proliferate at a future date.[7]

Third, the model only permits one-sided armament. If arms races are a form of a repeated prisoner's dilemma, then we presumably already have an explanation for arms treaties–neither proliferates because the other side will proliferate in response, in a manner similar to grim trigger strategies or tit-for-tat (Axelrod 1984; Wagner 2010, 175-176). Thus, any form of nonproliferation agreements must result from a different mechanism.[8]

And fourth, we make the declining state strategically vulnerable insofar as it must offer a division of the stakes to the rising state before the rising

effective deterrent threats.

[7]Notably, in these later models, the rising state's inability to proliferate occurs endogenously. It is trivial to show that rising states acquire nuclear weapons if proliferation is a now-or-never opportunity, but it is odd to *assume* that a rising state would suddenly lose the ability to proliferate, especially since such an outcome leads to a commitment problem and inefficiency.

[8]In fact, in Chapter 5, we will show that the mechanism described in the model sabotages tit-for-tat or grim trigger strategies in two-sided proliferation games precisely because of the attractiveness of butter-for-bombs deals.

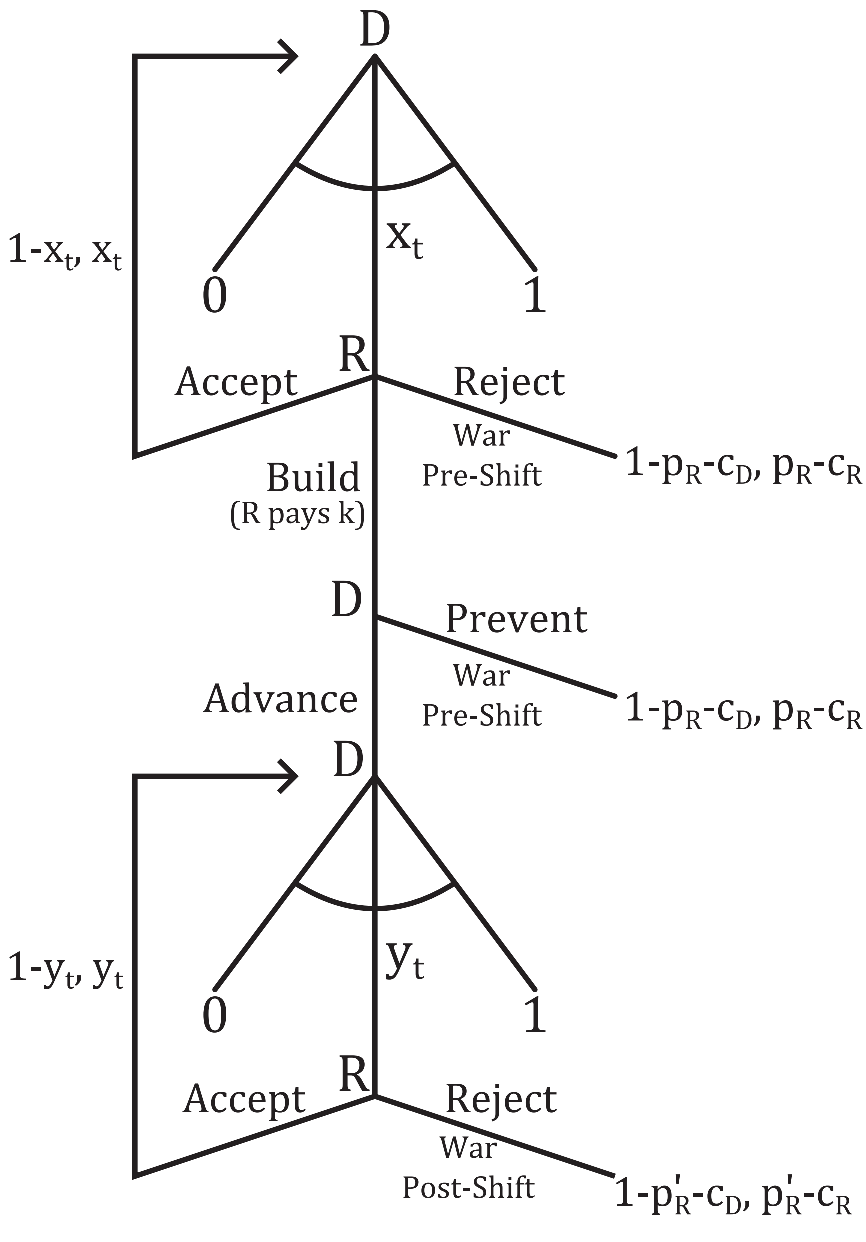

Figure 3.1: The model. All payoffs listed are for the period, though the war outcomes lock in their respective payoffs every period for the rest of time.

state chooses whether to build, and the declining state cannot retract that offer should the rising state proliferate. A major policy concern with Iran is that Iran could take the concessions the United States offers it, renege on any quid-pro-agreement not to build, and proliferate anyway. That being the case, the United States ought not to give any concessions, since any hypothetical bribe would not alter Iran's endgame behavior. Ordering the moves in this manner allows us to directly address the policy concern.

It is worth stressing that that structuring bargaining in this way means that the declining state does not directly negotiate over the rising state's weapons program. Given concerns regarding anarchy, making quid-pro-quo offers as in Chadefaux's (2011) model raises the question why rising states simply do not renege after receiving the concessions or why declining states do not renege after the rising state temporarily suspends its weapons program. Yet, interestingly, removing quid-pro-quo bargaining in favor of indirect bargaining does not stack the deck against cooperation.

In that vein, Debs and Monteiro (2013) analyze a similar model in which a rising state chooses whether to develop weapons programs in secret. However, their focus is on the how the secret nature of some weapons programs leads to preventive war. Thus, the structure of their game precludes analysis of negotiating over weapons.[9] We instead bring negotiations to the forefront. This allows the rising state to act more strategically, selecting to build only if its offers are insufficient. Knowing this, the declining state has incentive to offer large amounts upfront to deter the rising state to build. As we will see, this is exactly what occurs in equilibrium.

3.1.3 Equilibrium

Since this is a dynamic game with an infinite number of periods, we restrict our analysis to stationary Markov perfect equilibria (MPE):

[9]Specifically, in the finite version of their model, the the rising state chooses whether to build weapons at the beginning of each period. Cooperation is inherently impossible in such a setup because the rising state's decision dictates the period's terms of bargaining. If the rising state chooses not to build, then the declining state needs to only offer the rising state an amount to avert war in the pre-shift state of the world. Therefore, the rising state cannot threaten to proliferate if it receives poor offers, which in turn leads to the rising state receiving bare-bones concessions if it opts not to build at the start. As a result, the rising state must build at the beginning to receive substantial concessions at any point.

> **Definition**: A stationary Markov perfect equilibrium is a subgame perfect equilibrium in which players' strategies are only a function of the state of the world and are independent of calendar time.

In particular, the strategies players select in a stationary Markov perfect equilibrium cannot be a function of the history of players' strategies. However, the players' strategies can affect the state of the world, which can in turn affect strategy selection. In terms of this game, R's decision to build moves the actors into the post-shift state of the world if D does not prevent, and the actors' strategies will rightly change after the shift in the balance of power.

Before stating the main results, the following lemma will prove useful:

Lemma 3.1. *In every stationary MPE, in every post-shift period, D offers $y_t = p'_R - c_R$, and R accepts.*

The appendix provides a complete proof of Lemma 3.1. However, the intuition is a straightforward application of Fearon's seminal bargaining game. Since war creates deadweight loss to the system, D can always offer enough to satisfy R, and the optimal acceptable offer is preferable to war for D as well. Thus, D offers just enough to induce R to accept, and D keeps all of the surplus. In particular, R earns $p'_R - c_R$ and D earns $1 - p'_R + c_R$ for the rest of time, and peace prevails in the post-shift state of the world.

We are now ready for the first proposition.

Proposition 3.1. *If $p'_R - p_R > \frac{c_D + c_R}{\delta}$, D offers $x_t = p_R - c_R$ in the unique stationary MPE. R accepts these offers and never builds.*

Note that the left side of the inequality represents the extent of the power shift and the right side represents the inefficiency of war. When the shift is sufficiently greater than war's inefficiency, the power shift is "too hot." If R were to build, D would respond with preventive war. As a result, the threat of D's stick makes R's threat to build incredible. In turn, D can treat the bargaining problem as though R cannot build. Consequently, the declining state offers $x_t = p_R - c_R$ (the amount R would receive in a static bargaining game), R accepts, and the states avoid war.[10]

[10] The fact that preventive war does not occur here should be unsurprising since the game has complete information and power shift is observable and endogenous (Chadefaux 2011).

The appendix contains a complete proof. Intuitively, we derive the critical value of $p'_R - p_R$ by finding the value for which D prefers preventive war if it offers $x_t = p_R - c_R$ and R attempts to build:

$$1 - p_R - c_D > (1 - p_R + c_R)(1 - \delta) + \delta(1 - p'_R + c_R)$$
$$p'_R - p_R > \frac{c_D + c_R}{\delta}$$

This is the critical value of $p'_R - p_R$ presented in Proposition 3.1.

Proposition 3.2. *If $p'_R - p_R < \frac{k(1-\delta)}{\delta}$, D offers $x_t = p_R - c_R$ in the unique stationary MPE. R accepts these offers and never builds.*

Note the right side of the inequality reflects the time-adjusted cost of building. When the magnitude of the shift is too small relative to that cost, the power shift is "too cold" for the rising state to invest in weapons. D observes that R does not have a credible threat to build and therefore offers the same concessions it would offer if power were static. As a result, though for different reasons, the observable outcome for these parameters are the same as the outcome for Proposition 3.1's parameters.

The full proof appears in this chapter's appendix. We derive the critical value of $p'_R - p_R$ by finding the value for which the rising state will not build in response to $x_t = p_R - c_R$:

$$p_R - c_R > (p_R - c_R)(1 - \delta) + \delta(p'_R - c_R) - k(1 - \delta)$$
$$p'_R - p_R < \frac{k(1 - \delta)}{\delta}$$

This is the critical value of $p'_R - p_R$ presented in Proposition 3.2.

Before moving on, we maintain that $p'_R - p_R$ does not fall into the previously discussed cases and assume that $k \in (\frac{\delta(p'_R - p_R - c_D - c_R)}{1-\delta}, \frac{\delta p'_R - p_R}{1-\delta} + c_R)$. The minimum value constraint for k implies D earns more from engaging in a butter-for-bombs settlement than it does from earning its war payoff in the pre-shift stage. The maximum value constraint ensures that R never prefers rejecting to a successful power shift, even if D offers R nothing during the pre-shift periods. For the purposes of this chapter, these cases are theoretically uninteresting and offer no further insight to our analysis.

Proposition 3.3. *If $k > \delta(p'_R - c_R)$, D offers $x_t = p'_R - c_R - \frac{k(1-\delta)}{\delta}$ in all pre-shift periods in the unique stationary MPE; R accepts and never builds.*

Moving outside the parameters of Proposition 3.1 and Proposition 3.2 leaves the world "just right" for a power shift. Nevertheless, if the corresponding investment remains relatively costly, D prefers making immediate concessions. If R were to build in response, it would receive additional concessions in the post-shift state of the world. However, those additional concessions do not cover the costs of building, which in turn convinces R to accept the original offer. In effect, D manipulates R's opportunity cost for building to the point that investment is no longer profitable. On the other hand, if k is small, D prefers taking as much as it can at the beginning before acceding to the rising power afterward.

The appendix contains proof for Proposition 3.3. The equilibrium offer size $p'_R - c_R - \frac{k(1-\delta)}{\delta}$ equals R's continuation value for building, which is enough to induce R to accept. Note that D receives the remainder, or $1 - p'_R + c_R + \frac{k(1-\delta)}{\delta}$. For D to prefer taking that amount over the long-term to taking everything up front and suffering the consequences of proliferation later, it must be that the investment cost is relatively large. This generates the $k > \delta(p_R - c_R)$ requirement.

Before moving on, a couple comparative statics from Proposition 3.3 recur throughout this book, so it is worth briefly understanding them. First, as the extent of the power shift $(p'_R - p_R)$ increases, D's bribe increases. This might seem counterintuitive–a large power shift means that R is potentially *very* weak at the beginning. Yet R can use this exact weakness to its advantage because its outside option (investing in weapons) is correspondingly very desirable. As such, D must give larger concessions to induce R to accept.[11]

Second, D's offer decreases as k increases. That is, R receives better butter-for-bombs offers the smaller its investment cost is. Although engaging in butter-for-bombs deals means that R does not build, D knows it can keep more for itself and still induce compliance if the cost to invest is comparably more expensive.

We now move to our final proposition:

Proposition 3.4. *If $k < \delta(p'_R - c_R)$, D offers $x_t = 0$ in all pre-shift periods in the unique stationary MPE; R builds and D does not prevent.*

The proof and intuition follow from Proposition 3.3. Note that the minimalist butter-for-bombs offer $p'_R - c_R - \frac{k(1-\delta)}{\delta}$ increases as k decreases. Thus,

[11] This will be a recurring theme in the later chapters' case studies: a state feels vulnerable, threatens to proliferate, and extracts concessions. The weakness becomes a bargaining strength.

the remainder for D decreases as k decreases. Thus, if k is sufficiently small, D prefers taking everything upfront, letting R proliferate, and making larger concessions later on.

However, the as Chapter 5 discusses at length, the inefficiency benefits no one–both parties would be better off if k were higher and the states reached the equilibrium outcome Proposition 3.3 describes.

3.1.4 Numerical Example

To illustrate the logic of the butter-for-bombs equilibrium, consider the following specific environment. Let $p_R = .2$, $p'_R = .5$, $c_D = .3$, $c_R = .1$, $\delta = .9$, and $k = 1$. These values fit the parameters for Proposition 3.3.

If the game ever reaches the post-shift state, Lemma 3.1 states that D offers $x_t = p'_R - c_R = .5 - .1 = .4$ in every post-shift period. R accepts those offers and earns .4; D earns the remainder, or $1 - x_t = .6$.

In the pre-shift stage, if D offers $x_t = 0$, R earns 0 for accepting and $p_R - c_R = .2 - .1 = .1$ for rejecting. If R builds, D does not prevent, as it earns $1 - p_R - c_D = 1 - .2 - .3 = .5$ for preventing and $1 - \delta + \delta(1 - p'_R + c_R) = 1 - .9 + .9(1 - .5 + .1) = .64 = \frac{32}{50}$ for advancing to the next period. In turn, R earns $0 + \delta(p'_R - c_R) - k(1 - \delta) = .9(.5 - .1) - (1 - .9) = .26$ for building, which is more than it receives for rejecting or accepting.

Alternatively, if D offers $x_t = p'_R - c_R - \frac{k(1-\delta)}{\delta} = .5 - .1 - \frac{1(1-.9)}{.9} = \frac{13}{45}$, R earns $\frac{13}{45}$ for accepting. In contrast, R receives only .1 for rejecting and $\frac{13}{45}$ at most for building, so accepting is optimal. D earns $\frac{32}{45}$ for this outcome, whereas it receives only $\frac{32}{50}$ if it offers $x_t = 0$. Therefore, $x_t = \frac{13}{45}$ is the optimal offer for D.

Figure 3.2 illustrates the bargaining dynamics of this specific example drawn to scale, conceptualized as R and D negotiating over a strip of territory between their respective capitals. The top half represents the pre-shift balance of power. If the rising state rejects the offer or D prevents, the states fight to an expected outcome of p_R, but pay their respective costs c_R and c_D. The bottom half represents the post-shift balance of power. If the states fight here, the average outcome swings in R's favor to p'_R.

The dashed line at $p'_R - c_R - \frac{k(1-\delta)}{\delta}$ is the equilibrium outcome. R must receive more than the minimum acceptable amount ($p_R - c_R$) in a static bargaining game, otherwise it can profitably shift power and enjoy great concessions in the future ($p'_R - c_R$). However, D need not offer $p'_R - c_R$ to induce R not to build. Indeed, the equilibrium outcome sees R receive less

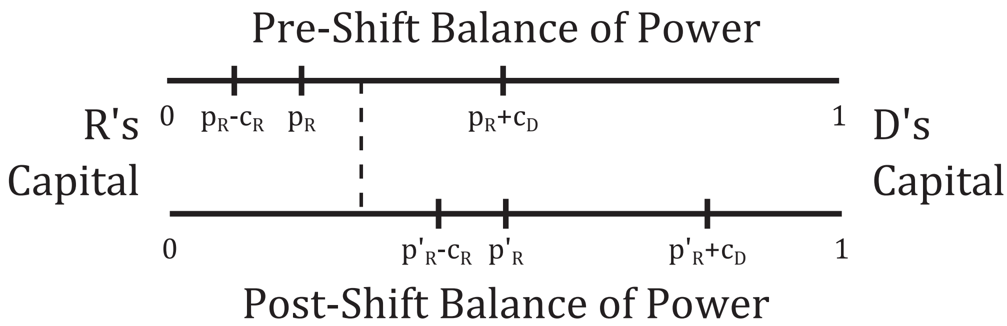

Figure 3.2: The pre-shift and post-shift balances of power. The figure is drawn to scale for the numerical example. In equilibrium, R receives $p'_R - c_R - \frac{k(1-\delta)}{\delta}$ (the dashed line) in every period, and D receives the remainder.

in every period than it would in the future if R actually shifted power. D effectively leverages the cost of building against R; the difference between $p'_R - c_R$ and the equilibrium outcome over time equals the discounted cost that R pays to build. The efficiency of the equilibrium outcome ensures D's satisfaction; because R never pays the inefficient cost k, D can extract it out of the negotiated settlement.

3.1.5 Robustness

As with any stylized model, it is worth asking whether the results are a function of the particular modeling choices or indicative of a broader underlying mechanism. The previous section showed the existence of a bargaining range. Any settlement within that range leaves both parties better off than a world with investment, and the rising state cannot profitably renege under those terms. Thus, the butter-for-bombs result is not sensitive to particular bargaining protocols. However, we may instead wonder about the reasonableness of some of the structural assumptions. In this subsection, we address a few possible issues.

Prestige. In the course of proliferating, many statesmen cite international "prestige" as a benefit to having nuclear weapons. Some researchers have shown concern regarding prestige as well (O'Neill 2006). While there are many reasons to be skeptical of the prestige argument[12], advocates might worry that the prestige negates the cost of proliferating k. Accordingly, k may drop to the critical value for which Proposition 3.4 predicts proliferation. In the extreme, k may even be negative.

Fortunately for the nonproliferation regime, this is a misinterpretation of the parameters. The cost parameter k only affects R's payoffs directly. However, prestige is zero sum. If *all* states had nuclear weapons, for example, then nuclear weapons would not be prestigious. As such, if nuclear weapons truly provide prestige, each additional state that proliferates drains prestige from the status quo nuclear powers.

While k does not have a zero sum interpretation, recall in contrast that p_R and p'_R refer to a zero sum bargaining good. In a world where nuclear weapons provide prestige, we could simply rescale the bargaining good to be the bargaining good *and* international prestige. Thus, prestige merely inflates the value of p'_R.[13] It does not render nonproliferation agreements impossible.[14] Put differently, if nuclear weapons shift prestige from status quo nuclear states to new proliferators, the status quo states ought to find a way to buy off potential proliferators and reap the benefits of the saved investment costs.

Punishment for Reneging. Consider off-the-equilibrium-path play in the butter-for-bombs parameter range. If R builds despite D's generous offer, it receives no punishment. Instead, R keeps everything D offered it for the period and then receives additional concessions once R obtains nuclear weapons. In effect, R is only rewarded for proliferating and faces absolutely no punishment for defying D. This was a deliberate modeling choice, as policymakers worry that immediate concessions leave declining states in this strategically vulnerable position. Even in this worst case scenario, butter-for-bombs agreements worked.

[12]For example, the nonproliferation regime has succeeded in making nuclear weapons a taboo source of military power. It is also difficult to disentangle actual beliefs about prestige from bargaining posturing.

[13]In that regard, it is clear why non-nuclear states claim that prestige exists while recognized nuclear weapons states claim the opposite. If it exists, declining states must concede more benefits to deter proliferation; if it does not, rising states receive no additional benefits.

[14]States might face a problem if prestige is indivisible, but even then they could negotiate side payments (in the form of the continuous bargaining good) to avoid inefficient outcomes.

Still, we may wonder what happens if D can sanction R in response to R building. Rather than assuming that R keeps its entire share of the offer if it builds, suppose instead that R only keeps $\alpha \in [0, 1)$ of the offer; equivalently, D recoups $1 - \alpha$ of the concessions.[15] Intuitively, if D attempts to buy R's compliance but fails, D can cut the remainder of the payment.

Following the proof for Proposition 3.3, R is willing to accept x_t if:

$$x_t \geq \alpha(1-\delta)(x_t) + \delta(p'_R - c_R) - (1-\delta)k$$

$$x_t \geq \frac{\delta(p'_R - c_R) - (1-\delta)k}{1 - \alpha(1-\delta)}$$

Note that this amount is strictly less than the amount D had to pay previously. This is unsurprising–if D can recoup a portion of its bribe, R finds investment less profitable and is therefore willing to accept a wider range of offers. Thus, butter-for-bombs settlements still exist and become easier to agree on under such conditions.

Prior Investment in Nuclear Research. Suppose R has the option to pre-invest (or sink) a certain amount of the cost of nuclear weapons before reaching the bargaining phase. The original model reveals the outcome of this modification. Depending on how much R pre-invests, two outcomes are possible. First, if the pre-investment is small, the remaining investment is sufficiently costly. Consequently, the states will reach a butter-for-bombs agreement. Note that, compared to the original model, R preforms better during the bargaining phase when it has already invested in nuclear weapons; with the remaining cost lower, D must give R a greater portion of the pie to successfully negotiate a butter-for-bombs settlement. However, R fares no better than before, as the improved payoff R receives in the bargaining phase equals upfront cost R pays. On the other hand, D fares substantially worse if R pre-invests because it must compensate R for the pre-investment. This compensation ultimately comes out of D's share of the butter-for-bombs bargain, leaving D in worse shape.

Second, suppose the pre-investment is large. Then D keeps the entire good for itself in the first period, R builds, and then D makes great concessions thereafter. Butter-for-bombs fails. But this only leads to more inefficiency,

[15] When $\alpha = 1$, the interaction is the original model.

since R finishes constructing the nuclear weapon in this case. Again, R does not profit from pre-investment, and D is strictly worse off.[16]

While international relations does not have a complete theory over how states choose to bargain[17], we can nevertheless conclude that it would be strange to reach either of these outcomes. Inefficiency in international relations is understandable when at least one party benefits or commitment problems prohibit mutually preferable alternatives. But here, inefficiency occurs for no discernable reason. D therefore has strong incentives to proactively engage R in negotiations, ensure R's investment cost remains as high as possible, and reach a butter-for-bombs deal. Failure to do so leads to unnecessary proliferation and Pareto inferior outcomes.

Bargaining over Objects that Influence Future Bargaining Power. In the model, the status quo division of the bargaining good does not effect military power in future periods. This caps the minimally acceptable butter-for-bombs at $p'_R - c_R - \frac{k(1-\delta)}{\delta}$, which in turn assures that R will not demand more than that in the period after the first butter-for-bombs agreement. However, if the division of the good affects power, one concern might be that D will refuse to offer concessions upfront–D might be afraid a butter-for-bombs deal will lead to a never-ending stream of increasingly large concessions.

Fortunately, this concern is not an issue. Fearon (1996) provides the intuition. He considers a model in which control over the bargaining good also determines military strength. In equilibrium, the states mitigate the shift in power by moderating each period's transition; the side receiving additional concessions accepts smaller offers at first, knowing the extra control of the good will allow it to coerce yet more concessions out of its rival in the future. As long as the good is infinitely divisible, war never occurs.

The analogous result would apply in the context of butter-for-bombs. D can offer R a smaller bribe upfront. R is willing to accept because it knows it can extort more later on, while D is satisfied because it receives a larger share earlier. The result remains efficient: R never proliferates, and both sides share the surplus.

[16] In this case, *both* are strictly worse off. Chapter 5 explores this point in greater depth.

[17] For progress along these lines, see Stone 2011 and Leventoglu and Tarar 2005.

Imperfect Monitoring. Debs and Monteiro (2013) focus on how D's imperfect observation of whether R is investing or not clouds the bargaining environment and leads to war. Thus, we may wonder whether butter-for-bombs agreements are sustainable if we relax the perfect information assumption.

Chapter 8 tackles this extension directly. However, for now, the simple answer is that butter-for-bombs agreements are resistant to imperfect monitoring. Why? D would not be willing to offer concessions if R would build after receiving them. But R chooses not to build here because it is simply not profitable to do so. Note that this has nothing to do with D's informational awareness. Thus, having imperfect information does not create an impediment to butter-for-bombs agreements.[18]

3.2 Interpretation

The butter-for-bombs equilibrium highlights the importance of *potential* power in regard to the stability of settlements. A sizeable literature in international relations debates whether systems with states of relatively equal power are more stable than systems where one state has a preponderance of power.[19] The rationalist literature critiques these theories by noting that the difference between relative power and relative benefits underlies incentives for war (Powell 1996; Reed et. al. 2008). In that regard, in the static bargaining model illustrated in Figure 3.2, any settlement on the interval $[p_R - c_R, p_R + c_D]$ is satisfactory to both parties.

Incorporating the possibility of shifting power changes the bargaining range, however.[20] D still must receive no less than its reservation value for pre-shift war, or $1 - p_R - c_D$. Previously, war was R's only outside option, which paid $p_R - c_R$. Here, building is a better outside option. Thus, R must now receive at least $p'_R - c_R - \frac{k(1-\delta)}{\delta}$ to not want to alter the status quo.

[18]In fact, Chapter 8 shows that butter-for-bombs deals expand to other parameter spaces, as D must pay a premium in the parameter space of Proposition 3.1–with imperfect information, D cannot leverage the stick of preventive war to coerce R not to build and consequently must provide concessions instead.

[19]Although the literature goes far beyond these two works, see Morgenthau (1960) for the balance of power argument and Blainey (1988) for the preponderance of power argument.

[20]CINC scores (Singer 1987), a standard metric of military power, include measures of population, iron and steel production, and energy consumption to evaluate unmobilized power. Our model instead indicates that *uninvested* power also influences the bargaining range.

Figure 3.3: R's equilibrium offer size as a function of p'_R, with the same parameters as Figure 3.2. When the shift is too small or too large, the rising state cannot credibly threaten to build and thus receives no concessions. In the middle range, the rising state's potential power coerces concessions, and its payoff is increasing in the extent of the potential shift.

Therefore, the range of stable settlements in which the states do not fight wars and power does not shift is the set $[p'_R - c_R - \frac{k(1-\delta)}{\delta}, 1 - p_R - c_D]$.

Figure 3.3 illustrates D's equilibrium offers in the pre-shift state as a function of p'_R, with the parameters held fixed as in the earlier numerical example. When $p'_R \in (\frac{1}{5}, \frac{14}{45})$, R cannot successfully recoup its building cost. D therefore treats the bargaining problem as though power were static and offers R its pre-shift reservation value for war, which R accepts. When $p'_R \in (\frac{14}{45}, \frac{29}{45})$, R can credibly threaten to build. D utilizes the butter-for-bombs bargaining tactic, which induces R to accept the immediate concessions and not build. Finally, when $p'_R \in (\frac{29}{45}, 1)$, R cannot credibly threaten to build if it receives its pre-shift reservation value for war, as D responds to building with preventive war. Consequently, D stands firm and still induces R to accept.

Further, Figure 3.3 illustrates R's non-monotonic preferences over future power. If the power shift is very small, the ability to build does not affect R's payoff at all. In the middle range, R can successfully threaten to shift

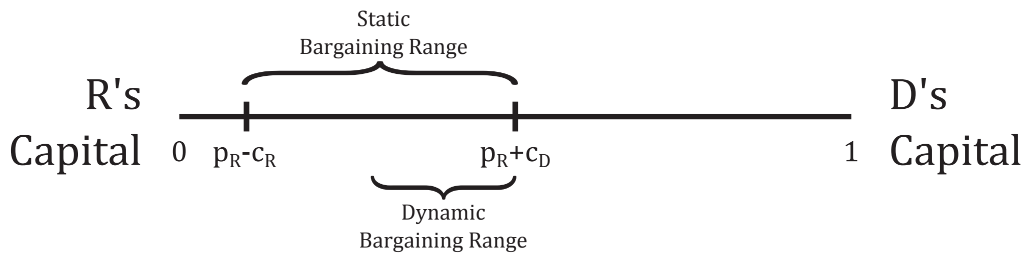

Figure 3.4: The set of Pareto settlements in a static bargaining game versus the dynamic bargaining game presented here.

power, which in turn causes D to make concessions. Moreover, these concessions are increasing in the extent of the power shift. However, the power shift eventually becomes too great, and R cannot successfully build without inducing D to intervene. Thus, R's payoff drops precipitously, as though R does not have the ability to shift power.

Finally, Figure 3.4 shows the set of stable outcomes for situations of static and dynamic power. If the rising state cannot build additional weapons, then any settlement on the interval $[p_R - c_R, p_R + c_D]$ is Pareto improving over war. If the rising state has access to weapons, then the range of settlements that Pareto dominate power shifts and war is $[p'_R - c_R - \frac{k(1-\delta)}{\delta}, p_R + c_D]$. Note that this is a subset of the Pareto dominant set in the static world.

This causes problems for an outside observer trying to understand which game the states are playing. If the observer recorded an outcome on the interval $[p_R - c_R, p'_R - c_R - \frac{k(1-\delta)}{\delta})$, then she knows the states are in a static environment, otherwise the rising state could increase its payoff by shifting power. However, if the outcome is on the interval $[p'_R - c_R - \frac{k(1-\delta)}{\delta}, p_R + c_D]$, the observer cannot differentiate between a world in which the rising state cannot shift power and a world in which the rising state simply does not want to. Time will not resolve this problem either; since the rising state's potential power is sufficient to extract the concessions, it never builds the weapons and never demonstrates the dynamic nature of power in the interaction.

3.3 Implications of Butter-for-Bombs Agreements

During his time in office, U.S. President John F. Kennedy feared a world of perhaps twenty-five nuclear states.[21] And by 1964, five states (the United States, the Soviet Union, the United Kingdom, France, and China) held nuclear arsenals, perhaps signalling the dawn of a global nuclear age. Yet, since the Nuclear Non-Proliferation Treaty's (NPT) creation in 1968, 190 countries have signed the treaty, and only North Korea has ever withdrawn. Meanwhile, Israel, South Africa, India, and Pakistan are the only other countries to have tested a nuclear bomb.[22] So, at least thus far, the world has not reached the nuclear tipping point that Kennedy feared.

Yet functional nuclear weapons provide inherent security and allow states to coerce additional concessions out of their rivals during times of crisis (Beardsley and Asal 2009; Kroenig 2013). In light of this, why haven't more states followed in North Korea's footsteps by withdrawing from the NPT and joining the nuclear club?

The model provides a causal explanation: nuclear weapons are simply not in high demand in the context of a bargaining game. Bargaining is constant-sum; if nuclear weapons provide indirect benefits to their possessors, then they must also indirectly hurt their possessors' antagonists. Consequently, those fearing proliferation have incentive to offer attractive deals to shut down the nuclear contagion. Meanwhile, the potential proliferator has incentive to listen. After all, nuclear weapons are far from free. Those states would happily accept most of what they hope to gain from proliferating without investing in an actual nuclear test.

Figure 3.5 shows why demand is so low. When nuclear weapons cause too great of a power shift relative to the declining state's costs of intervention, the rising state declines to proliferate so as to avoid preventive war. Here, the declining state need not offer any carrots to induce compliance, as its stick is a sufficient threat to deter the rising state. Moreover, deterrence gives the declining state its best possible outcome, as it does not have to resort to costly war. In other words, declining states need not use carrots when sticks are credible.

As a result, a state seeking a nuclear arsenal must first shore up its conven-

[21] Reiss 2004, 4.

[22] Of these, South Africa dismantled its weapons at the end of Apartheid.

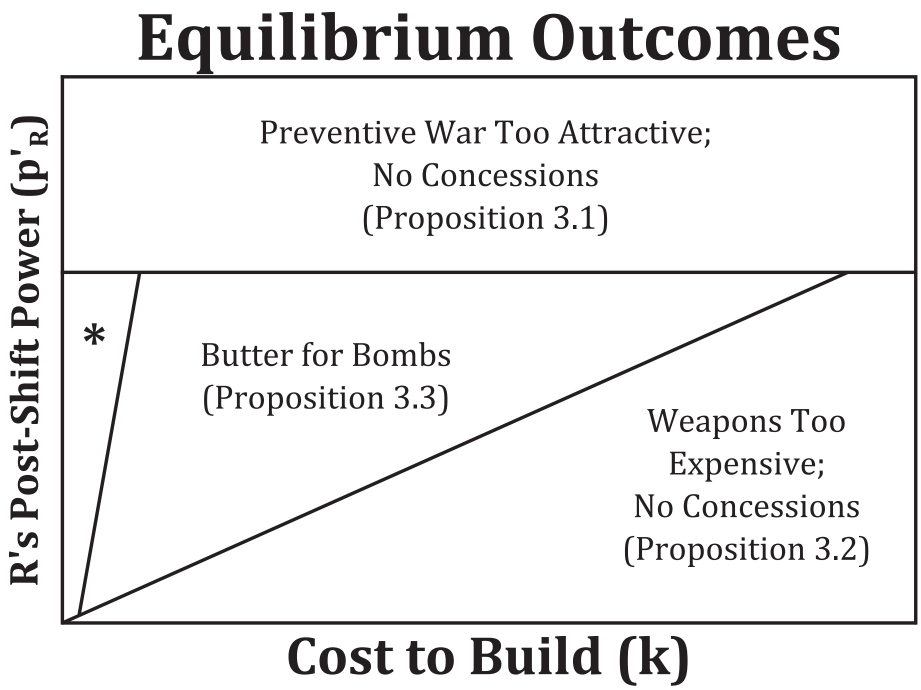

Figure 3.5: Equilibrium outcomes as a function of the cost to build and the rising state's level of future power. Investment only occurs in the region containing the asterisk, as described in Proposition 3.4.

tional deterrent, otherwise proliferation is not a strategically viable option. Iran has correspondingly placed many of its nuclear facilities underground. This location limits the damage from a possible aerial strike, reducing the Israeli or American ability to effectively intervene.

But even if the potential proliferator can defend itself from an invasion, it still might not want to seek nuclear weapons. After all, bombs are an investment in the future. Such an investment is only sensible if it yields sufficient returns. Thus, states will not proliferate if the financial cost of nuclear weapons is too great. Moldova or Rwanda might view nuclear weapons as attractive in theory. However, the cost to proliferate would bankrupt those countries before they could achieve a nuclear capacity. Similarly, states need to have a contentious security issue for proliferation to make sense. Iceland and Ireland have the technical know-how and financial resources to build a bomb, but it is unclear what sort of benefits said bomb could bestow.

Proliferation remains an ineffective strategy even as the attractiveness of the investment increases. At this point, the potential rising state is conditionally willing to shift power. But it is in the declining state's best interest to bribe the would-be nuclear state and avoid facing the consequences of a much stronger rival. The states ultimately resolve the crisis without proliferation, as the immediate concessions ensure that building a bomb will not lead to a better outcome.

The model also reveals that bargaining over nuclear weapons does not require rising state to commit to the incredible. In negotiating over nuclear weapons programs, many commentators (Bolton 2010, Krauthammer 2009, and Fly and Kristol 2010) have warned that potential proliferators cannot be bought off. That being the case, declining states should hold their ground, as bribes have no effect on tomorrow's power politics.

Yet the model shows that such a strategy creates a self-fulfilling prophecy. Standing firm in the present causes rising states to redouble their efforts. Forestalling negotiations therefore creates the exact nuclear problem declining states wish to avoid. Resolving conflict requires parties to have the correct incentives. Here, with no other bargaining frictions present, it is remarkably easy to obtain a rising state's compliance. The declining states can always buy off their competitors. It is just a question of whether those declining states prefer impatiently hoarding as much of the benefit for as long as possible to conceding a modest amount throughout time.

As a final note, these findings instruct us to take a holistic approach to understanding nuclear proliferation. Quantitative studies frequently attempt to understand proliferation behavior by analyzing "supply side" components of nuclear weapons (Meyer 1985; Jo and Gartzke 2007); we ought to expect states with limited nuclear capacities to not develop nuclear bombs. While the model (via Proposition 3.2) confirms the value of supply side explanations, we cannot limit our focus purely to nuclear capacity. Indeed, Figure 3.5 shows that supply side arguments explain outcomes in the bottom right portion of the parameter space only; preventive war and bargaining determine the remaining outcomes.

Ignoring these other factors leads to strange interpretations of the data. Sagan (2011, 229-230) notes that, according to the Jo and Gartzke (2007) estimates, Trinidad and Tobago "had a higher degree of nuclear weapons latency in 2001 than is North Korea, which was only five years away from detonating its first nuclear weapon." But latent capacity does not become active capacity without the will of the state. Trinidad and Tobago has no significant coercive bargaining relationship and maintains an active military force in the thousands. Meanwhile, North Korea has technically been at war since the 1950s and has more than a million active duty soldiers. Thus, latency measures requires context. We must look at bargaining relationships to understand why states ultimately choose to develop nuclear capacity.

3.4 Conclusion

This chapter formally investigated the credibility of butter-for-bombs settlements. Although international relations scholars traditionally emphasize how fully realized power extracts concessions, the model demonstrated that *potential* power is sufficient. Declining states have incentive to proactively bargain with rising states, so as to ensure that nuclear non-proliferation remains the status quo. Rising states have incentive to welcome the offers, as they can obtain most of their goals without paying costs to develop a weapons program. Credible non-proliferation agreements result.

The model makes a significant contribution to our understanding of costly weapons production. At present, our explanations for non-armament are limited to the threat of preventive war and inefficient investments; we lack a model that explains how carrots convince states to forgo weapons programs. The butter-for-bombs model fills the gap, showing how states can manipulate their rivals' opportunity cost and thereby avoid nuclear proliferation.

While the model reveals the absence of commitment problems and the existence of bargaining space, it fails to provide any intuition as to exactly how states reach butter-for-bombs deals. Consequently, the next chapter provides case studies to corroborate the usefulness of the model. Later chapters then add bargaining frictions–shifting resolve, third-party sources of nuclear materials, incomplete information, and imperfect information–to study whether butter-for-bombs agreements remain credible in these contexts.

3.5 Appendix

3.5.1 Proof of Lemma 3.1

Existence: Suppose there exists a stationary MPE in which x_t^* is offered in every period. First, we show that R must accept $x_t > p'_R - c_R$. Let $V(accept)$ be R's continuation value for accepting. Since R can always reject and lock in $p'_R - c_R$ for the rest of time, it follows that $V(accept) \geq p'_R - c_R$. Thus, accepting $x_t > p'_R - c_R$ is strictly better than rejecting if:

$$(1-\delta)x_t + \delta V(accept) > p'_R - c_R$$

Using $V(accept) = p'_R - c_R$ as a lower bound, we can show this by demonstrating the following:

$$(1-\delta)x_t + \delta(p'_R - c_R) > p'_R - c_R$$

$$x > p'_R - c_R$$

This holds, so R must accept.

Now we show that R must reject $x_t < p'_R - c_R$. Rejecting earns R $p_R - c_R$ for the game. Thus, R must reject if:

$$p'_R - c'_R > (1-\delta)x_t + \delta(V(accept))$$

Suppose this did not hold. Then it must be that R accepts because $V(accept) > p_R - c_R$. For $V(accept)$ to be that large, D must offer R more than $p'_R - c_R$ in each period. But if R optimally accepts when $x_t < p'_R - c_R$, it cannot be optimal for D to offer more than $p'_R - c_R$ since D's payoff is strictly decreasing in x_t and R is willing to accept a smaller amount. Given that, R must reject.

The above proof implies that R could accept or reject if $x = p'_R - c_R$. To prove existence of the equilibrium, assume R accepts with probability 1 when indifferent.

Now suppose x_t^* is a stationary MPE strategy for D. If $x_t^* > p'_R - c_R$, then R accepts in every period, and D earns $1 - x_t^*$. However, D could profitably deviate to $x_t = \frac{x_t^* + p'_R - c_R}{2}$ in period t. R still accepts. This is a profitable deviation if the following holds:

$$(1-\delta)(1 - \frac{x_t^* + p'_R - c_R}{2}) + \delta(1 - x_t^*) > 1 - x_t^*$$

$$x_t^* > p'_R - c_R$$

This holds. Thus, x_t^* cannot be greater than $p'_R - c_R$.

If $x_t^* < p'_R - c_R$, then R rejects, leading to an absorbing state in which D earns $1 - p'_R - c_D$. However, D could make a one-shot deviation to $x_t = p'_R$ for the period. R accepts, and D earns $1 - p'_R$ for that period and its war payoff $1 - p'_R - c_D$ for the rest of time. Thus, this is a profitable deviation, as $1 - p'_R > 1 - p'_R - c_D$.

Finally, if $x_t^* = p'_R - c_R$, then R accepts these offers, and D earns $1 - p'_R + c_R$. D cannot make any profitable deviations. If it deviates to $x_t < p'_R - c_R$ in a period, R rejects, and D earns $1 - p'_R - c_D$ for that period and the rest of time, which is less than what it earns for maintaining the strategy and earning $1 - p'_R + c_R$ instead. Meanwhile, if D offers $x_t > p'_R - c_R$ in a period, R still accepts. However, D earns strictly less for that period and maintains its same payoffs for the rest of time, so this is not a profitable deviation. Thus, the strategies in Lemma 1 are a stationary MPE. □

Uniqueness: The remaining case is to show that R cannot accept with probability $q \in [0, 1)$. For proof by contradiction, suppose such a stationary MPE existed. Then in any period of the post-shift state, D earns $1 - p'_R + c_R$ with probability q and $1 - p'_R - c_D$ for the rest of time with probability $1 - q$. Thus, D receives a payoff of:

$$q(1 - p'_R + c_R)(1-\delta) + (1-q)(1 - p'_R - c_D)$$

$$+q\delta[q(1 - p'_R + c_R)(1-\delta) + (1-q)(1 - p'_R - c_D)]$$

$$+q^2\delta^2[q(1 - p'_R + c_R)(1-\delta) + (1-q)(1 - p'_R - c_D)]$$

$$+...$$

This is a geometric series with discount rate $q\delta$ and base $[q(1 - p'_R + c_R)(1 - \delta) + (1 - q)(1 - p'_R - c_D)]$. Therefore, we can write D's expected utility as:

$$\frac{q(1 - p'_R + c_R)(1 - \delta) + (1 - q)(1 - p'_R - c_D)}{1 - q\delta}$$

There exists a one-shot profitable deviation in period t if there exists an $x'_t > p_R - c_R$ such that the following condition holds:

$$(1 - x'_t)(1 - \delta) + \delta \left(\frac{q(1 - p'_R + c_R)(1 - \delta) + (1 - q)(1 - p'_R - c_D)}{1 - q\delta} \right)$$

$$> \frac{q(1 - p'_R + c_R)(1 - \delta) + (1 - q)(1 - p'_R - c_D)}{1 - q\delta}$$

Substantial algebraic manipulation eventually yields the following:

$$x'_t < p'_R + \frac{c_D(1 - q) - qc_R(1 - \delta)}{1 - q\delta}$$

So a profitable deviation exists if there exists an x'_t such that:

$$p'_R - c_R < x'_t < p'_R + \frac{c_D(1 - q) - qc_R(1 - \delta)}{1 - q\delta}$$

One exists if:

$$p'_R - c_R < p'_R + \frac{c_D(1 - q) - qc_R(1 - \delta)}{1 - q\delta}$$

$$(c_D + c_R)(1 - q) > 0$$

This holds, since $q < 1$. Therefore, a profitable deviation exists, meaning no stationary MPE exists in which R accepts with probability $q \in [0, 1)$ when indifferent between accepting and rejecting. □

3.5.2 Proof of Proposition 3.1

Existence: Suppose there exists a stationary MPE in which D offers x^*_t. Consider how R must behave in response to $x_t > p_R - c_R$. R cannot build when $x_t \geq p_R - c_R$. If R does, D prevents if:

$$1 - p_R - c_D > (1 - x_t)(1 - \delta) + \delta(1 - p'_R + c_R)$$

Note that because $x_t \geq p_R - c_R$ in this case, we have $(1 - p_R + c_R)(1 - \delta) + \delta(1 - p'_R + c_R) \geq (1 - x_t)(1 - \delta) + \delta(1 - p'_R + c_R)$. Therefore, we may show that preventing is optimal for D by instead demonstrating the following inequality:

$$1 - p_R - c_D > (1 - p_R + c_R)(1 - \delta) + \delta(1 - p'_R + c_R)$$

$$\delta > \frac{c_D + c_R}{p'_R - p_R}$$

This inequality holds for Proposition 1. R earns $p_R - c_R - k(1 - \delta)$ for this outcome. However, R could reject and earn $p_R - c_R$ instead, a profitable deviation.

In choosing between accept and reject, R must accept if:

$$x_t(1 - \delta) + \delta V(accept) > p_R - c_R$$

Similar to Lemma 1, R is always capable of rejecting and locking in a payoff of $p_R - c_R$. Thus, it must be that $V(accept) \geq p_R - c_R$. Using $p_R - c_R$ as a lower bound, R must accept if $x_t > p_R - c_R$. R could accept or reject if $x_t = p_R - c_R$. Assume for now that R accepts with probability 1.

If $x_t < p_R - c_R$, it cannot be the case that R accepts. Rejecting and earning $p_R - c_R$ is better for R if:

$$p_R - c_R > (1 - \delta)x_t + \delta V(accept)$$

For this to fail to hold, it must be that $V(accept) > p_R - c_R$. But this cannot be the case in equilibrium. If R accepts $x_t < p_R - c_R$, D would unwilling to offer larger amounts, as D's payoff is strictly decreasing in x_t. D also would not be willing to offer an amount that induces R to build either. R would only build if it produced a payoff greater than $p_R - c_R$. But since bargaining is constant sum, if R earns more than $p_R - c_R$ from bargaining, it must be the case that D earns less than $1 - p_R + c_R$. Yet D could offer an acceptable $x_t < p_R - c_R$ and earn more. Therefore, R cannot accept $x_t < p_R - c_R$.

In turn, there can be no stationary MPE in which D offers $x_t^* > p_R - c_R$, as D could make a one-time deviation in period t to $x_t = \frac{x_t^* + p_R - c_R}{2}$ and still induce R to accept. This gives D a strictly greater payoff for period t while maintaining its same payoffs for the rest of time.

There also can be no stationary MPE in which D offers $x_t^* < p_R - c_R$. R must reject or build. If R rejects in equilibrium, D earns $1 - p_R - c_D$. However, D could profitably deviate by offering $x_t = p_R$ in period t. This induces R to accept and gives D a strictly greater payoff for period t while maintaining the same payoffs for the rest of time. If R builds in equilibrium, if D responds by preventing, D earns the same war payoff as before and could profitably deviate to $x_t = p_R$ in period t.

Lastly, D cannot make an equilibrium offer which induces R to build if D does not prevent. For R to want to build in this situation, the following must hold:

$$x_t^*(1-\delta) + \delta(p'_R - c_R) - k(1-\delta) \geq p_R - c_R$$

$$x_t^* \geq \frac{p_R - c_R}{1-\delta} - \frac{\delta(p'_R - c_R)}{1-\delta} + k$$

But D has a profitable one-shot deviation in period t if it offers $p_R - c_R + \epsilon$ (with $\epsilon > 0$) and the following holds:

$$(1 - x_t^*)(1-\delta) + \delta(1 - p'_R + c_R)$$

$$> (1 - p_R + c_R - \epsilon)(1-\delta) + \delta[(1 - x_t^*)(1-\delta) + \delta(1 - p'_R + c_R)]$$

$$x_t^* < \frac{p_R - c_R}{1-\delta} - \frac{\delta(p'_R - c_R)}{1-\delta} + \frac{\epsilon}{1-\delta}$$

Stringing together the last two inequalities shows the values of x_t^* for which those strategies are mutually optimal:

$$\frac{p_R - c_R}{1-\delta} - \frac{\delta(p'_R - c_R)}{1-\delta} + k \leq x_t^* < \frac{p_R - c_R}{1-\delta} - \frac{\delta(p'_R - c_R)}{1-\delta} + \frac{\epsilon}{1-\delta}$$

A profitable deviation exists if there exists a value for ϵ such that:

$$\frac{p_R - c_R}{1-\delta} - \frac{\delta(p'_R - c_R)}{1-\delta} + k < \frac{p_R - c_R}{1-\delta} - \frac{\delta(p'_R - c_R)}{1-\delta}$$

$$\epsilon > k(1-\delta)$$

The value k is strictly positive. Thus, we can always find an ϵ that gives D a profitable deviation. Consequently, there can be no stationary MPE in which D offers $x_t < p_R - c_R$, R builds, and D does not prevent.

In turn, if $\delta > \frac{c_D + c_R}{p'_R - p_R}$, D offers $x_t = p_R - c_R$ every period, R accepts these offers, and the game continues peacefully in the stationary MPE. □

This leaves $x_t^* = p_R - c_R$ as the only option. R accepts and D earns $1 - p_R + c_R$. If D deviates to a larger x_t, R still accepts, but this is a needless concession for D. If D deviates to a smaller x_t, R rejects or builds. In either scenario, D earns more from offering $x_t^* = p_R - c - R$. □

Uniqueness: For proof by contradiction, suppose R could accept with probability q in a stationary MPE. Then in any period of the pre-shift state, D earns $1 - p_R + c_R$ with probability q and $1 - p_R - c_D$ for the rest of time with probability $1 - q$. But these are the same payoffs as the uniqueness proof for Lemma 1, except we have replaced p'_R with p_R. Nevertheless, continuing with that substitution, all further steps of the uniqueness proof for Lemma 1 apply. Therefore, no other stationary MPE exist. □

3.5.3 Proof of Proposition 3.2

Existence: Suppose there exists a stationary MPE in which D offers x_t^*. Begin by noting that R cannot build regardless of D's offers for these parameters. If R builds, it earns $p_R - c_R - k(1-\delta)$ if D prevents in response and $x_t(1-\delta) + \delta(p'_R - c_R) - k(1-\delta)$ if D does not. In the first case, R could profitably deviate to reject in period t and earn $p_R - c_R$, which is strictly better than receiving the same payoff but paying $k(1-\delta)$. In the second case, if $x_t \geq p_R - c_R$, R could make a one-shot deviation to accepting in period t. This is a profitable deviation if:

$$x_t(1-\delta) + \delta V(accept) > x_t(1-\delta) + \delta(p'_R - c_R) - k(1-\delta)$$

Recalling that $V(accept)$ must be at least $p_R - c_R$, we can show the above inequality holds by instead showing:

$$x_t(1-\delta) + \delta V(accept) > x_t(1-\delta) + \delta(p'_R - c_R) - k(1-\delta)$$

Since $x_t \geq p_R - c_R$, this holds if:

$$p_R - c_R > p'_R - c_R - \frac{k(1-\delta)}{\delta}$$

$$\delta < \frac{k}{k + p'_R - p_R}$$

This holds for Proposition 2. Moreover, note that this implies that R must accept if $x_t > p_R - c_R$, since rejecting gives R a payoff of $p_R - c_R$ while accepting gives R $x_t > p_R - c_R$ for the period and a continuation value of at least $p_R - c_R$ for the rest of time.

If $x_t < p_R - c_R$, building remains suboptimal. Instead, R could make a one-shot deviation to rejecting in period t. This is a profitable deviation if:

$$p_R - c_R > (p_R - c_R)(1 - \delta) + \delta(p'_R - c_R) - k(1 - \delta)$$

$$\delta < \frac{k}{k + p'_R - p_R}$$

Again, this holds for Proposition 2.

Meanwhile, R cannot accept if $x_t < p_R - c_R$. It earns $(1-\delta)x_t + V(accept)$ for this outcome. This can only be as good as rejecting if $V(accept) > p_R - c_R$, but D cannot be willing to make offers larger than $p_R - c_R$ if R is willing to accept some $x_t < p_R - c_R$.

Thus, R accepts if $x_t > p_R - c_R$, rejects if $x_t < p_R - c_R$, and could accept or reject if $x_t = p_R - c_R$. As usual, assume that R accepts with probability 1 in this case for now.

Now suppose $x_t^* > p_R - c_R$. R must accept. However, D could make a one-shot profitable deviation by offering $x_t = \frac{x_t^* + p_R - c_R}{2}$ instead in period t. R must still accept. In turn, this is profitable for D if:

$$\left(1 - \frac{x_t^* + p_R - c_R}{2}\right)(1 - \delta) + \delta(1 - x_t^*) > 1 - x_t^*$$

$$x_t^* > p_R - c_R$$

So x_t^* cannot be greater than $p_R - c_R$.

Moving on, suppose $x_t^* < p_R - c_R$. R rejects here. But D could make a one-shot deviation to $x_t = p_R$ in period t. This is profitable if:

$$(1 - p_R)(1 - \delta) + \delta(1 - p_R - c_D) > 1 - p_R - c_D$$

$$c_D > 0$$

.

Since this holds, x_t^* cannot be less than $p_R - c_R$.

This leaves $x_t^* = p_R - c_R$ as the only remaining possibility. R accepts, and D earns $1 - p_R + c_R$. D cannot profitably deviate once to any $x_t < x_t^*$ in period t; this causes R to reject, changing D's payoff to $1 - p_R - c_D$. Likewise, D cannot profitably deviate once to any $x_t > x_t^*$ in period t. Although R still accepts, D earns x_t for the period instead of x_t^* while maintaining the same future payoffs, which is strictly worse.

Therefore, in the stationary MPE, D offers $x_t^* = p_R - c_R$, and R accepts. □

Uniqueness: We must now show that R cannot accept with probability $q \in [0, 1)$ and reject with complementary probability.[23]

For proof, suppose R could accept with probability q in a stationary MPE. Then in any period of the pre-shift state, D earns $1 - p_R + c_R$ with probability q and $1 - p_R - c_D$ for the rest of time with probability $1 - q$. Once again, these are the same payoffs as the uniqueness proof for Lemma 1, except we have replaced p'_R with p_R. Nevertheless, continuing with that substitution, all further steps of the uniqueness proof for Lemma 1 apply. Therefore, no other stationary MPE exist. □

3.5.4 Proof of Proposition 3.3

Existence: Suppose there exists a stationary MPE in which D offers x_t^*. Begin by finding R's response to an offer x_t in a pre-shift period. R cannot reject any $x_t < p'_R - c_R - \frac{k(1-\delta)}{\delta}$. To see this, suppose R rejected such an x_t. Consider a one-shot deviation to building in period t. The parameters for Proposition 3 imply that D does not prevent in such a scenario. Therefore, the deviation is profitable for R if:

$$\delta(p'_R - c_R) - k(1 - \delta) > p_R - c_R$$
$$k < \frac{\delta p'_R - p_R}{1 - \delta}$$

[23] We can rule out the possibility of R building as a knife-edge condition. That is, such a case requires R's war payoff to be exactly equal to R receiving its pre-shift war payoff in the first period and its post-shift share of the bargain minus its costs, or:

$$\frac{p_R - c_R}{1 - \delta} = p_R - c_R + \frac{\delta(p'_R - c_R)}{1 - \delta} - k$$
$$p'_R = p_R + k$$

However, this is a knife-edge condition by definition.

This holds for Proposition 3's parameters. So R cannot reject $x_t < p'_R - c_R - \frac{k(1-\delta)}{\delta}$.

Moreover, R cannot accept $x_t < p'_R - c_R - \frac{k(1-\delta)}{\delta}$. Again, suppose not, and let $V(accept)$ be R's continuation value for accepting under x^*. Then R is only willing to accept if:

$$(1-\delta)x_t + \delta V(accept) \geq (1-\delta)x_t + \delta(p'_R - c_R) - k(1-\delta)$$

$$V(accept) \geq p'_R - c_R - \frac{k(1-\delta)}{\delta}$$

R cannot obtain that value in a stationary MPE through accepting offers from D; if R were accepting here, then D could make an offer $x_t < p'_R - c_R - \frac{k(1-\delta)}{\delta}$ which R is still willing to accept but is strictly better for D. So R must obtain that value from building. But in that case, D's continuation value must be no more than $1 - (p'_R - c_R - \frac{k(1-\delta)}{\delta})$, which means D earns more by making an offer $x_t < p'_R - c_R - \frac{k(1-\delta)}{\delta}$ that R accepts. So R must build in response to $x_t < p'_R - c_R - \frac{k(1-\delta)}{\delta}$.

R cannot reject $x_t > p'_R - c_R - \frac{k(1-\delta)}{\delta}$. Note that because R can always reject and lock-in a payoff of $p_R - c_R$, it must be that $V(accept) \geq p_R - c_R$. Using the equality as a lower bound, accepting is better than rejecting if:

$$(1-\delta)x_t + \delta(p'_R - c_R) > p_R - c_R$$

$$x_t > p_R - c_R$$

Recall that $x_t > p'_R - c_R - \frac{k(1-\delta)}{\delta}$. Thus, the inequality holds if:

$$p'_R - c_R - \frac{k(1-\delta)}{\delta} > p_R - c_R$$

$$p'_R - p_R > \frac{k(1-\delta)}{\delta}$$

This is true. Therefore, R cannot reject $x_t > p'_R - c_R - \frac{k(1-\delta)}{\delta}$.

Alternatively, R could build in response. This is not optimal for R if D prevents in response, since R could lock in its war payoff by rejecting and not pay the cost to build. If D advances to the next period, R accepts if:

$$(1-x_t) + \delta(p'_R - c_R) - k(1-\delta) > x_t(1-\delta) + \delta V(accept)$$

$$V(accept) \geq p'_R - c_R - \frac{k(1-\delta)}{\delta}$$

Otherwise, R builds.

Now consider D's decision. It cannot be the case that $x_t^* > p'_R - c_R - \frac{k(1-\delta)}{\delta}$. D could make a one-shot deviation to offering the midpoint between that x_t and x_t^* in period t. Since this offer is still strictly greater than $p'_R - c_R - \frac{k(1-\delta)}{\delta}$, R must accept. This is a profitable deviation for D, as D keeps slightly more in period t and maintains its same payoff in all future periods.

It also cannot be the case that $x_t^* \in (0, p'_R - c_R - \frac{k(1-\delta)}{\delta})$. R builds in response. But D can make a one-shot deviation in period t to offering $x_t = 0$. R still builds and D still does not prevent, so D earns the same payoff for all future periods. However, D keeps slightly more in period t, which makes this a profitable deviation.

Therefore, x_t^* must be 0 or $p'_R - c_R - \frac{k(1-\delta)}{\delta}$. If D offers $x_t = 0$, R builds, D does not prevent, and D earns $1 - \delta + \delta(1 - p'_R + c_R)$. If D offers $x_t = p'_R - c_R - \frac{k(1-\delta)}{\delta}$, R accepts, and D earns $1 - p'_R + c_R + \frac{k(1-\delta)}{\delta}$. In turn, D optimally offers $p'_R - c_R - \frac{k(1-\delta)}{\delta}$ if the following holds:

$$1 - p'_R + c_R + \frac{k(1-\delta)}{\delta} > 1 - \delta + \delta(1 - p'_R + c_R)$$

$$k > \delta(p'_R - c_R)$$

So if these parameters hold, in the unique stationary MPE, D offers $x_t^* = p'_R - c_R - \frac{k(1-\delta)}{\delta}$ in the pre-shift periods, and R accepts.[24] □

Uniqueness: If the game falls under the parameters of Proposition 3 and $k > \delta(p'_R - c_R)$, the body of the paper proved that D must offer $p'_R - c_R - \frac{k(1-\delta)}{\delta}$ in every stationary MPE. This section shows that there are no stationary MPE in which R does not accept with probability 1.

For proof by contradiction, suppose there existed a stationary MPE in which R accepted with probability $q \in [0, 1)$ and built with complementary probability. Similar to the proof for the uniqueness of Lemma 1, D's expected utility is a geometric series with discount rate $q\delta$ and base payoff of:

$$q\left[\left(1 - p'_R + c_R + \frac{k(1-\delta)}{\delta}\right)(1-\delta)\right]$$

[24] By analogous argument, if $k < \delta(p'_R - c_R)$, D offers $x_t^* = 0$ in the pre-shift periods in the unique stationary MPE, thereby forcing R to build. These are the strategies listed in Proposition 3.4.

$$+(1-q)\left[(1-p'_R+c_R+\frac{k(1-\delta)}{\delta})(1-\delta)+\delta(1-p'_R+c_R)\right]$$

Let $[*]$ be the sum of this geometric series. D can profitably deviate to some $x'_t > p'_R - c_R - \frac{k(1-\delta)}{\delta}$, which forces R to accept with probability 1, in period t if the following condition holds:

$$(1-x'_t)(1-\delta)+\delta[*] > [*]$$
$$x'_t < 1-[*]$$

In turn, such a value exists if:

$$p'_R - c_R - \frac{k(1-\delta)}{\delta} < x'_t < 1-[*]$$
$$p'_R - c_R - \frac{k(1-\delta)}{\delta} < 1-[*]$$

Substantial algebraic manipulation eventually yields $k(1-\delta) > 0$. This holds. As such, if R accepts $x_t = p'_R - c_R - \frac{k(1-\delta)}{\delta}$ with probability $q \in [0,1)$, D has a profitable deviation. Consequently, there exists no stationary MPE in which R accepts $x_t = p'_R - c_R - \frac{k(1-\delta)}{\delta}$ with probability $q \in [0,1)$ and therefore the equilibrium presented in Proposition 3 is unique. □