Testing the Robustness of the Bargaining Model of War

Due Beginning of Class February 10, 2015

No Late Work Accepted

In class, we saw how the costs of war ensure that a range of settlements exist that are mutually preferable to war. This homework assignment tests the robustness of the model to see whether such a range exists under looser assumptions.

To orient the assignment and introduce the common notation you should use throughout this problem set, recall the basic model. Suppose two rival unitary actors exist: R(ebels) and G(overnment). They can resolve their conflict through war or negotiated settlement. If they fight, R prevails with probability p_R and receives all 1 of the good, while G receives 0. G prevails the remaining $1 - p_R$ portion of the time and receives all 1 of the good while R receives 0. Either way, the actors pay respective costs $c_R > 0$ and $c_G > 0$. Thus, R's expected payoff for fighting equals:

$$p_R(1) + (1 - p_R)(0) - c_R$$

$$p_R - c_R$$

And G's expected value equals:

$$p_R(0) + (1 - p_R)(1) - c_G$$

$$1 - p_R - c_G$$

Alternatively, they could divide the good peacefully, in which case R receives x and G receives $1 - x$. A side is satisfied with peace if the settlement produces at least as large of a payoff as its value for war. Consequently, R finds the settlement acceptable if:

$$x \geq p_R - c_R$$

And G finds it acceptable if:

$$1 - x \geq 1 - p_R - c_G$$

$$x \leq p_R + c_G$$

Using the previous two inequalities, a division x is mutually acceptable if:

$$p_R - c_R \leq x \leq p_R + c_G$$

And such an x exists if:

$$p_R - c_R \leq p_R + c_G$$

$$c_R + c_G \geq 0$$

This is true, so peaceful settlements always exist.

The following questions weaken the assumptions of the above model. Please replicate the above proof under those weaker assumptions. Unless otherwise noted, maintain the same notation as above.

Group work is encouraged. However, each problem set write up must be done individually. Show all work.

Note that the additional assumptions do not "stack" as the problem set progresses, i.e., discard the new assumptions from question 1 as you do question 2 and discard the new assumptions from question 2 as you do question 3.

1) **Multiple Outcomes.** Before, all wars ended in complete victory or complete defeat. Suppose instead that R wins and takes the entire good with probability p_R, G wins and takes the entire good with probability p_G, and an international intervention occurs with probability $1 - p_R - p_G$. If the intervention occurs, a caretaker government is established that gives .6 of the good to G and .4 of it to R. (In essence, the government achieves a very minor victory.) Regardless of the outcome, states always pay their war costs as normal.

a. Write each side's expected value for war. (One point.)

b. Prove that a mutually preferable peaceful alternative x always exists. (One point.)

2) **Uncertainty.** A common misconception is that uncertainty over the likely outcome of conflict will lead to bargaining breakdown. This is not true when both sides face the same uncertainty. Imagine a scenario where the likelihood of victory depends heavily on military cohesion. Yet there is no way to know how cohesive the troops are without actually fighting. Even so, both the rebels and government believe that the rebels will be cohesive with probability q and uncohesive with probability $1 - q$. In the first case, the rebels win with probability p_R; in the second, they with with probability p'_R. (Essentially, cohesion is good for the rebels' probability of victory.) Regardless of the cohesion, the actors pay the same costs as before.

a. Write each side's expected value for war. (One point.)

b. Prove that, despite the uncertainty, a mutually preferable peaceful alternative x always exists. (One point.)

c. In class, we saw that *asymmetric* uncertainty about the probability of victory can lead to war. Briefly explain why asymmetric uncertainty creates bargaining problems but symmetric uncertainty does not. (Two points.)

3) **Private benefits.** The previous cases all used the unitary actor assumption. This time, suppose the government is a unitary actor but the rebels have a leader who controls his group's decision to go to war. The leader still internalizes the cost c_R as before. However, if the rebels win, he derives some private benefit $b > 0$ from being in charge of the government. (This could be because it will boost his ego, give him a longer page on Wikipedia, or ensure a lifetime of steak dinners.) Despite the leader's bias for war, the purpose of this question is to show that peaceful agreements can still work provided that b is not too large.

a. Write the government's and the rebel leader's expected values for war. (One point.)

b. What is the maximum value of b such that a peaceful settlement still exists? Hint: Attempt to prove the existence of a peaceful settlement as normal. Then rewrite the final inequality in terms of b. (One point.)

c. Multiply the inequality in part 3b by p_R and interpret its meaning substantively. That is, explain what it represents in English without using any mathematical symbols. (Two points.)