# Cornering the Market: Optimal Governmental Responses to Competitive Political Violence

William Spaniel*

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**Abstract**

I develop a model that endogenizes entry into the market of competitive political violence. In equilibrium, an existing group may overproduce violence to capture all potential supporters and deter entry by a potential competitor. Contrary to some hypotheses about outbidding, violence can therefore be greater with only a single group than when a second group enters the market. I then investigate four manners by which a target government might mitigate the violence: offensive measures that undermine the lead group's marginal cost of violence, defensive measures that absorb a portion of all violence, deterrent measures that increase the cost of group formation, and concessions to the group's audience to reduce grievances. Of these, only defensive measures are guaranteed to decrease violence; increasing the burden of entry and decreasing grievances can counterintuitively *increase* violence.

*Assistant Professor, Department of Political Science, University of Pittsburgh. (williamspaniel@gmail.com, http://williamspaniel.com).

# 1 Introduction

Over the last decade, scholars of terrorism and insurgency have come to see group competition as a major motivation for political violence (Cronin 2011, 40). Rather than thinking of attacks as a message entirely for the target, organizations can also use violence to communicate with each other and audiences sympathetic to their ideology. Correspondingly, the theory of outbidding conceptualizes violence as a means through which groups compete for recruits and resources (Crenshaw 1985; Kydd and Walter 2006). Violence serves as an advertisement, and greater competition forces each group to spend more to broadcast their message. This literature often concludes that as the number of active groups increases, so does this competition, and thus so does the violence produced (e.g., Bloom 2005, 95).

That said, critics wonder about the generality of outbidding. Moghadam (2008, 36), for instance, notes that the Tamil Tigers began employing especially violent suicide missions in 1987, after direct competition had subsided. Furthermore, some of the most deadly terrorist and insurgent groups have been relatively hegemonic during their peak violence periods: Euskadi Ta Askatasuna (ETA), the Irish Republican Army, al-Qaeda in the late 1990s and early 2000s, and the Islamic State in the mid 2010s. Correspondingly, large-n statistical analyses have found an inconsistent relationship between groups and violence, with some scholars finding a positive relationship and others finding no relationship.[1]

In this paper, I argue that the theory of outbidding needs to expand beyond *observed* competition to properly understand the phenomenon. Organizations in the market for political violence must fear not only present competitors but also competitors on the horizon. Consequently, I develop a model of competitive political violence between an existing group and a potential entrant into the market. In equilibrium, existing organizations have incentive to saturate the market in violence, capture available resources, and deny potential competitors a profitable entry. Surprisingly, this strategy of *cornering the market* can be more violent than when groups actively compete with one another. The theory of outbidding therefore does not guarantee that more observed groups implies more violence, which makes sense of the contradictory empirical findings.

[1] See Clauset et al 2010; Findley and Young 2012; Stanton 2013; Nemeth 2014; Jaeger et al 2015; Fortna 2015; Conrad and Greene 2015.

While this finding is interesting in its own right, the model also produces a number of empirical and policy implications. Governments often wish to reduce violence targeted against them and are willing to pay costs to achieve that goal. Policymakers have suggested or tried at least four different strategies to accomplish this: (1) increasing the cost of group formation, (2) decreasing grievances among those who might support an organization, (3) launching offensive measures to destroy the infrastructure of existing groups, and (4) hardening potential targets of attacks.[2] I show that one cannot assess the effectiveness of counterterrorism and counterinsurgency strategies without understanding the second-order effects of group competition. Indeed, only one of these strategies is assuredly effective, and two may counterintuitively *increase* the prevalence of violence if the target does not fully commit to them. These discrepancies help explain inconsistencies in the empirical literature on counterterrorism (Lum, Kennedy, and Sherley 2006).

First, increasing the cost of group formation can backfire. When the costs of formation are low, cornering the market looks unattractive for the lead group. This is because it is easy for its competitors to enter, thereby requiring the lead group to exert too much effort to deter others from entering. When the costs of formation are high, however, the lead group finds cornering the market attractive. Because cornering can result in more violence than the competitive equilibrium, overall violence can increase at this transition point. Violence ultimately declines as the entry cost grows sufficiently large, though. Thus, the target may have to fully commit to the strategy to see any positive effects.

Second, decreasing grievances among those who might support an organization can also lead to an increase in violence. The logic is similar to the previous case. When the number of supporters of political violence is low, the market is less conductive multiple competing groups. This incentivizes the lead group to corner the market. When the number of supporters is large, the lead group may permit competitors to enter. As before, because competitive markets can have less violence overall, shifting from the competitive market to the cornered market can spike the level of violence. This helps explain the unclear relationship between reducing grievances and violence that the literature has previously uncovered (Brancati 2006; Dugan and Chenoweth

[2]See, for example, the National Strategy for Combating Terrorism (http://2001-2009.state.gov/s/ct/rls/wh/71803.htm), Pillar 2001 (29-40), and Netanyahu 1995 (132-147).

2012), where violence increases after concessions. As before, however, fully committing to reducing grievances eventually leads to a decrease in violence.

Third, I show that defensive measures are an assuredly effective strategy, as they lead to a guaranteed reduction in violence. Because defensive measures do not target a single group, they reduce violence from all sources. In turn, and in contrast to the previous two cases, defensive measures do not make cornering the market more attractive. And regardless of whether the lead group corners the market or the groups compete, the defensive measures counter a portion of overall violence as intended.

Finally, offensive measures to destroy the infrastructure of existing groups has weakly positive effects. Such tactics increase the amount of effort necessary for a lead group to produce violence. When this shifts the lead group's preference from cornering the market to allowing competition, overall violence decreases; this strategy does not lead to a spike like in the previous two cases because it directly reduces the lead group's violence output even when the market is competitive. It similarly leads to lower overall violence when another group would have entered the market in the absence of an intervention. However, if the lead group would corner the market without an intervention and still prefers to corner given the (possibly minimal) intervention, violence remains static. This is because the lead group calculates its level of violence to deter competition, and the intervention does not alter a competitor's payoff for entry.

The optimal offensive strategy has a second key policy implication. If the target wishes to offensively intervene against a cornering group, it must do so in a manner that leads to other groups successfully competing. Critics of the "War on Terror" argue that such measures are ineffective because they resemble "Whac-a-Mole"—knock one terrorist group down, and another simply springs up in its place. The model validates this replacement effect. However, critics overlook how debilitating the lead group disincentivizes the overly violenct cornering behaviors. Furthermore, it also cripples that lead group in competition with its newly formed rival. All told, the amount of violence the target stops the lead group from producing is greater than the added violence from the new group. Thus, whack-a-terrorist-group can be effective even if it spawns new political violence organizations.

# 2 A Model of Cornering and Competition

This section starts the analysis with a game featuring a pair of violent non-state actors; later, I will add a target government that can manipulate competition between them. I begin by describing the sequence of play, then I solve for the equilibrium and analyze the prevalence of violence across the observable outcomes.

## 2.1 The Game

**Players.** The two-player baseline model contains a pair of violent non-state actors. Group 1 is an existing producer of violence seeking to keep its support flowing. Group 2 is a potential entrant into the market that might compete for Group 1's resources.[3] I feature only two groups for transparency in the results. Nevertheless, it is worth stressing that the fundamental theoretical results I present are robust to interactions with multiple existing groups and multiple potential groups deciding whether to enter the market. This is because cornering the market to deny competitors entry allows all existing groups to share a larger pie, potentially making the existing groups willing to overproduce violence.

**Actions and Timing.** As Figure 1 illustrates, play begins with Group 1 choosing a level of violence $v_1 \geq 0$. Larger values represent greater effort exerted, which in turn leads to more violence against the target government that I will introduce later. Group 2 observes Group 1's selection and then decides whether to enter the market or not. Entering the market costs $c \in (0, 1)$, which represents the fixed costs of creating the organization.[4] If Group 2 enters, it selects a level of violence of its own $v_2 \geq 0$. The game ends.

**Preferences and Payoffs.** I use a contest success function to map group effort into a division of audience resources. Consequently, Group 1 earns $\frac{v_1}{v_1+v_2} - m_1 v_1$, where

[3]Although I refer to these as distinct groups, one may conceive of the potential entrant as an entrepreneurial individual within the existing group who is considering whether to splinter off.

[4]I constrain $c$ below 1 because the value of benefits is standardized to 1. Thus, if $c > 1$, Group 2 would want to stay out under all conditions. The rest of the analysis would be trivial and not be of theoretical relevance, so I omit it.

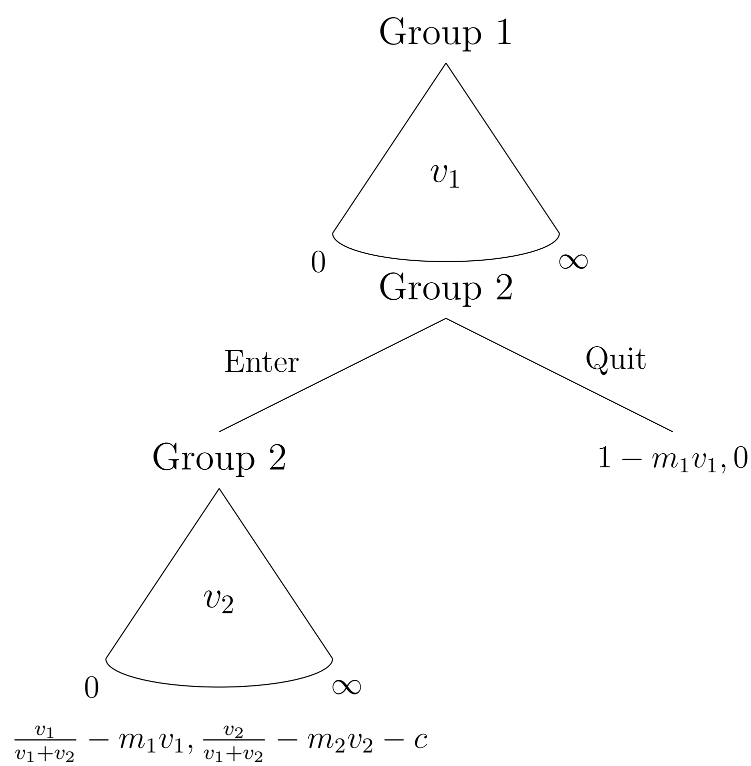


Figure 1: The two-player extensive form game.

$m_1 > 0$ represents Group 1's marginal cost for producing violence.[5] Note that if Group 2 does not enter, $v_2 = 0$ and therefore Group 1's payoff simplifies to $1 - m_1 v_1$. Group 2's payoff is similar, but it pays $c > 0$ to enter. Thus, it earns $\frac{v_2}{v_1+v_2} - m_2 v_2 - c$ if it enters, where $m_2 > 0$ represents its marginal cost of violence. This simplifies to 0 if Group 2 stays out.

Note that this setup clarifies the groups' motives as maximizing marketshare. This is the underlying motivation in the outbidding literature (Bloom 2005) and reflects the idea that some terrorist groups have preferences beyond policy change (Cronin 2011, 40). It also shows how groups can credibly threaten to use costly violence even in the absence of political goals. Including such preferences into the utility functions does not substantially alter the results presented below. Specifically, it does not change the instances in which a cornered market features more violence than a competitive market, nor does it affect how interventions into the market can increase or decrease outbidding violence.[6]

## 2.2 Solving for the Optimal Levels of Violence

Because this is a sequential game of complete information, I search for its subgame perfect equilibria (SPE). SPE refines Nash equilibrium by ensuring that all threats are credible—i.e., actions are optimal given the history of the game.

Although the interaction has few moves, it is complex to fully solve for due to the lack of restrictions on quantities of violence that the actors can produce. I therefore discuss the intuition behind each decision one step at a time. Moreover, I emphasize the intuition for these choices; the appendix contains full proofs wherever applicable.

### 2.2.1 Group 2's Violence Decision

Group 2's violence decision is straightforward. At this stage, Group 1 has already selected its level of violence, and Group 2 has already paid the cost of entry. It therefore

[5]Contest success functions like this are undefined when $v_1 + v_2 = 0$. This is not problematic, however, because regardless of how the benefits are divided, either party could profitably deviate to some arbitrarily small amount and capture the entire quantity.

[6]It does, however, lead to more violence overall than compared to this model. This is because the marginal cost of violence is effectively lower if more violence is more likely to lead to a favorable policy change.

only needs to optimize its payoff for the contest. Group 2's objective function for the contest is $\frac{v_2}{v_1+v_2} - m_2 v_2$. Optimizing this yields $v_2^* \equiv \sqrt{\frac{v_1}{m_2}} - v_1$.

Note that this optimal level of violence is decreasing in Group 2's marginal cost $m_2$; the more expensive violence is for Group 2, the less Group 2 is inclined to commit violence.[7] In addition, if Group 1's allocation is sufficiently large, the Group 2's marginal utility for each unit of violence begins to decline. Intuitively, producing one unit of violence produces a greater return for Group 2 when Group 1 has produced one unit of violence than when Group 1 has produced one million units. In fact, if Group 2's marginal cost and Group 1's level of violence are too high (i.e., $m_2 v_1 > 1$), Group 2 produces $v_2 = 0$.

#### 2.2.2 Group 2's Entry Decision

There are two cases to consider here. First, suppose that combination of Group 1's violence and Group 2's marginal cost is sufficiently high. Then Group 2 optimally produces $v_2 = 0$ and receives a payoff of 0 from the contest. If it enters under these circumstances, its payoff is $-c$. If it quits, it receives 0 instead. Group 2 therefore quits—it makes no sense to pay fixed costs of entry and then not exert any effort.

Second, suppose that the combination of Group 1's violence and Group 2's marginal cost is sufficiently low. Now Group 2 will optimally produce a positive amount of effort, namely $v_2^*$. It may nevertheless not wish to play the contest if the cost of entry is too great. Specifically, recall that Group 2's overall utility for entering and then choosing $v_2^*$ equals $\frac{v_2^*}{v_1+v_2^*} - m_2 v_2^* - c$. Group 2 therefore enters if this amount is greater than 0, its payoff for quitting. Setting up this inequality, substituting $v_2^* = \sqrt{\frac{v_1}{m_2}} - v_1$ and solving for $v_1$ yields:

$$v_1 > v_1^* \equiv \frac{(1-\sqrt{c})^2}{m_2}$$

Thus, Group 2 quits if $v_1 > v_1^*$, enters if $v_1 < v_1^*$, and is indifferent between the two if $v_1 = v_1^*$. As Group 1 produces more violence, Group 2's payoff for competition goes down. In turn, if Group 1's production is sufficiently high, Group 2 prefers quitting to

[7]For example, if the potential entrant cannot easily resolve principal-agent problems, its marginal cost of violence would be relatively high. Group 1 can exploit this weakness by producing less violence to corner the market.

paying a cost to enter a competition that will not end well. But if Group 1's production is sufficiently low, it is worth paying the cost of entry to obtain the benefits from the contest.[8] Note as well that Group 2 is less likely to enter when the cost of entry or its marginal cost is high.

### 2.2.3 Group 1's Violence Decision: To Corner or to Compete

Given that Group 2 only enters if $v_1$ is small enough, Group 1 can control Group 2's entry; producing large amounts of violence corners the market, while choosing a smaller value of $v_1$ induces competition. Although the appendix shows that the precise details of the strategy are more involved, Group 1's decision-making process basically takes the following form:

1. Calculate the minimum amount of violence production ($v_1$) such that Group 2 would want to quit. This is $v_1^*$.[9] (Choosing anything above the minimum is an unnecessary expense for Group 1, as Group 2 is already staying out and Group 1 is taking all of the benefits.)

2. Calculate Group 1's payoff for "cornering the market" in the manner described in (1).

3. Calculate the optimal value of $v_1$ conditional on Group 2 entering, which is $\frac{m_2}{4m_1^2}$.

4. Calculate Group 1's payoff for choosing the strategy in (3).

5. Compare the payoffs from (2) and (4). If (2) is larger, play (1); if (4) is larger, play (3).

Whether the payoff from (2) is larger than the payoff from (4) depends on $c$. Specifically if $c > c^* \equiv \left(1 - \sqrt{\frac{m_2}{m_1} - \frac{m_2^2}{4m_1^2}}\right)^2$, Group 1 prefers to corner the market. This is

[8] This critical value for $v_1$ provides insight into situations with multiple potential entrants. Suppose those entrants have differing costs of entry $c_i$ and $m_i$. Then the maximum of $\frac{(1-\sqrt{c_i})^2}{m_i}$ is enough to corner the market. This is because if the group with the highest expected utility for competing is unwilling to enter the market, all others would stay out as well. It is also true that violence spikes when Group 1 produces that amount instead of the optimal quantity to compete this best opposing group, and this spike drives all the key results of the extensions I develop below.

[9] This assumes that Group 2 quits when indifferent between entering and quitting. The appendix shows that equilibrium conditions require this.

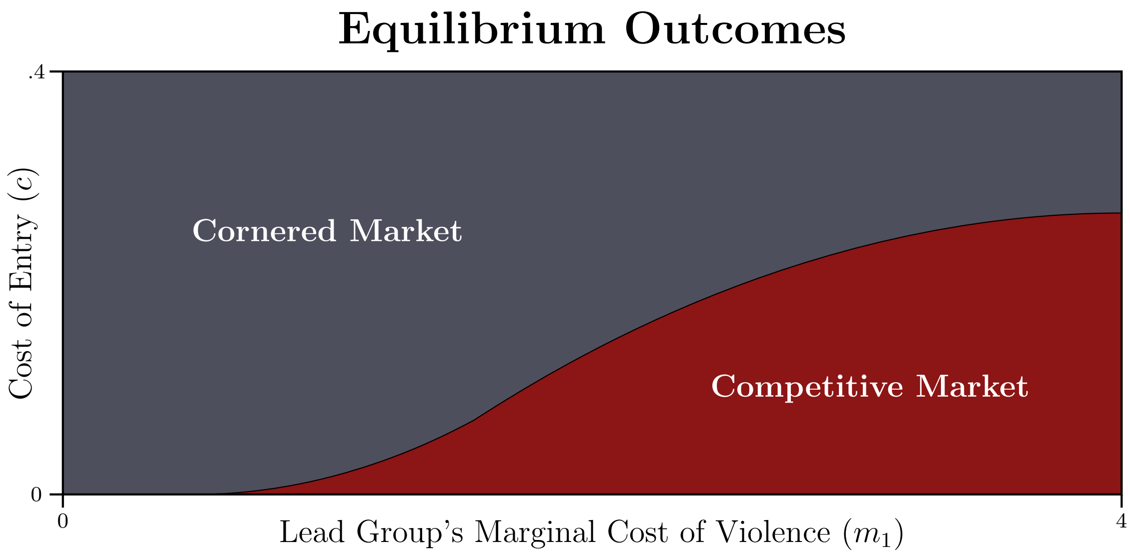


Figure 2: Substantive outcome of the game's equilibrium, with $m_2 = 1$. The lead group chooses to corner the market when its marginal cost of violence is low and the competitor's cost of entering is high.

because higher costs of entry mean that Group 1 can produce less violence and still deter Group 2 from entering. Cornering the market in turn looks more attractive, causing Group 1 to be more likely to pursue that strategy.

Meanwhile, if $c < c^*$, Group 1 finds cornering the market to be too expensive. In turn, Group 1 produces the optimal amount of violence as if it were expecting Group 2 to enter, and Group 2 indeed enters.

## 2.3 When Is Violence Most Prevalent?

Outbidding would seem to predict that violence peaks when multiple groups compete for their audience's resources. The model shows this intuition is not guaranteed to hold. There are only two outcomes equilibrium outcomes to compare: cornered and competitive markets. When Group 1 corners, it produces $v_1^* = \frac{(1-\sqrt{c})^2}{m_2}$ to convince Group 2 to not enter. This is the sum total of violence because Group 2 produces nothing when it stays out. In contrast, in a competitive market, Group 1 produces $\frac{m_2}{4m_1^2}$ and Group 2 responds with $\sqrt{\frac{\frac{m_2}{4m_1^2}}{m_2}} - \frac{m_2}{4m_1^2}$. Violence is greater in the first case if:

$$\frac{(1-\sqrt{c})^2}{m_2} > \frac{m_2}{4m_1^2} + \left(\sqrt{\frac{\frac{m_2}{4m_1^2}}{m_2}} - \frac{m_2}{4m_1^2}\right)$$

$$m_1 > \frac{m_2}{2(1-\sqrt{c})^2}$$

As the extensions below visualize and the appendix proves, this can hold for certain parameter values while still inducing the proper equilibrium outcome. In fact, it is true right at the boundary: when Group 1 just barely prefers cornering the market, overall violence is greater than when Group 1 just barely prefers inducing competition. Increasing $c$ to sufficiently large levels eventually makes the equilibrium quantity of violence lower with the cornered market. This is because Group 2 is relatively unwilling to enter; thus, Group 1 can commit a low level of violence to convince its opponent to stay out.

Why does violence peak with only one group? A numerical example illustrates the logic. Suppose that $m_1 = m_2 = 1$ and that Group 2 would assuredly enter. Then Group 1 optimally selects $\frac{m_2}{4m_1^2} = \frac{1}{4}$. Group 2 responds by selecting $\sqrt{\frac{v_1}{m_2}} - v_1 = \frac{1}{4}$. Because both select the same amount of violence for these parameters, they split the contest and each receive $\frac{1}{2}$.[10] After subtracting out their costs, each receives a final payoff of $\frac{1}{4}$, with a sum violence of $\frac{1}{2}$.

Note that out of the whole value of 1, Group 1 only nets a quarter of it. Consequently, Group 1 is willing to commit up to $\frac{3}{4}$ in violence to push Group 2 out of the market; this would give Group 1 the entire pie and leave it with more leftover than if it had competed. Put differently, it is willing to use more violence by itself to corner the market than the groups' combined violence in a competitive market. And, indeed, this is exactly what happens for some parameter spaces. For example, if $c = \frac{1}{25}$, then Group 1 can produce $v_1 = \frac{(1-\sqrt{c})^2}{m_2} = \frac{16}{25}$ in violence to deter Group 2's entry. This leaves Group 1 with a payoff of $1 - \frac{16}{25} = \frac{9}{25}$, which is better than its payoff inducing entry. As such, Group 1 corners the market by choosing a level of violence well above what both groups would produce together when in competition.

These results help explain the inconsistent relationship between groups and violence

[10] The fact that both produce the same amount here is a quirk of these particular parameters and does not generally occur.

that empirical studies of outbidding have uncovered.[11] Knowing how many groups exist is insufficient to understand the relationship between group numbers and violence. A market with only a single group may outproduce (*ceteris paribus*) a competitive market because that one group needs to deter entry of others. Potential groups are equally important. But even knowing that information still can lead to mixed results depending on the research design; although conditions exist where the single group outproduces competitive markets, a single group may produce less than a large competitive market if the barriers to entry are great.

The results also suggest a need to think more holistically about competition. Kydd and Walter (2006, 78), for example, argue that "one solution to the problem of outbidding would be to eliminate the struggle for power by encouraging competing groups to consolidate into a unified opposition." The model demonstrates an important hidden assumption underlying that claim: if consolidation creates new opportunities for other groups to enter the market, unification may backfire. Indeed, these types of manipulations can have strange effects on the market for competitive violence. I therefore formally extend the model below to allow for interventions.

# 3 Manipulating the Market

If targets of violence attempt to manipulate competitive incentives, they have a variety of strategies to choose from: (1) they may increase the barriers of entry for groups considering whether to form; (2) they can shrink the pool of potential support by reducing grievances among the affected population; (3) they may increase the marginal cost of violence for existing groups by taking offensive measures to gut those organizations; and (4) they can increase defensive measures to mitigate violence from all producers.

To understand how these strategies influence equilibrium play, I introduce a third player to the game: the Target. The Target begins the new interactions by choosing to pay a cost to shift these parameters in ways that would apparently handicap the groups in some way. But the Target must be careful. The results below indicate that these strategies can backfire, leading to *more* violence, not less. Intuitively, this because the new barriers can shift a competitive market into a cornered market, which is more

[11] Clauset et al 2010; Findley and Young 2012; Stanton 2013; Nemeth 2014; Jaeger et al 2015; Fortna 2015; Conrad and Greene 2015.

violent.

Before beginning, a word of caution is appropriate. Although these categories are useful for conceptualizing counterterrorism and counterinsurgency strategies, certain tactics may affect multiple components simultaneously. For example, broad-scope domestic communications surveillance can increase the difficulty of both planning an attack and coordinating the formation of an organization. Readers therefore ought to exercise caution in translating these results to specific policy recommendations, noting that the actual affect might be a combination of these multiple extensions.[12]

## 3.1 Increasing the Barriers to Entry

First, consider the cost of the competing group to enter, $c$. Targets can influence this cost in a number of ways. One major component of the National Strategy for Combatting Terrorism is to deny organizations sanctuary in rogue states. As such rogue states crumble, would-be groups must seek asylum in less desirable and more remote locations, increasing the burden of the initial outlay to establish an organization. Meanwhile, the September 11 attacks created a push to create a norm against terrorism. This process began with United Nations Security Council Resolution 1373, which instructs countries to codify anti-terrorism laws and ratify anti-terrorism conventions. Political violence entrepreneurs face increased hurdles in coordination the formation of a group under such conditions. Improving economic conditions can raise the opportunity cost of abandoning the civilian sector as well (Blomberg, Hess, and Weerapana 2004).

To incorporate these efforts by potential victims of violence into the model, suppose that the Target begins the game by choosing $c \in [\underline{c}, \infty)$. The value $\underline{c} > 0$ represents the cost of entering without any intervention by the Target. Let the Target's payoff have two components. First, it suffers the sum of the violence produced. Second, it pays a cost that is a function of how much effort the Target expends to increase the barrier of entry. The appendix fully solves the game in which this specific utility function equals $-(v_1 + v_2) - \alpha(c - \underline{c})$, where $\alpha$ measures how much the Target values effort versus violence; smaller values of $\alpha$ reflect a greater fear of violence, as the Target finds the per-unit increase of changing the cost to be less important. The marginal cost could

[12] For example, if the Target wanted to simultaneously alter $c$ and $m_1$, it would need to check that the new values of $c$ and $m_1$ it sets actually lead to less violence than with the undisturbed values of those quantities.

also reflect delay, with new barriers that require greater time to come online effectively having a larger $\alpha$. Regardless, the key results follow as long as the effort function is convex.[13]

Broadly, the Target's optimal strategy can take one of two forms. Recall that the cost of entry $c$ partially determines whether Group 1 corners the market. If $c$ is great, Group 1 deters Group 2 from entering because doing so only requires a modest amount of violence; if $c$ is small, on the other hand, Group 1 prefers competing with Group 2 because cornering requires too much violence. The value $c^* = \left(1 - \sqrt{\frac{m_2}{m_1} - \frac{m_2^2}{4m_1^2}}\right)^2$ reflects this cutpoint. Thus, as long as $\underline{c} < c^*$, the Target's decision determines whether Group 1 will corner or compete.[14]

The fact that the Target can turn a competitive market into a cornered market leads to the following result:

**Proposition 1.** *Increasing the fixed costs of entry (c) can lead to an increase in violence.*

Figure 3 helps communicate the logic. When $c < c^*$, the parties commit a fixed amount of violence in a competitive market. The precise value of $c$ does not impact the groups' production choices because, conditional on entering the market, the cost $c$ is sunk for Group 2 and therefore does not affect its violence decision. Pushing past $c^*$, however, spikes the violence because the market shifts from competitive to cornered; Group 1 now overproduces to exclude Group 2. Further increases $c$ decrease equilibrium violence on this range because Group 1 can produce less violence to convince Group 2 stay out as the cost of entry increases.

The shape of the violence levels has a number of important implications for counterterrorism and counterinsurgency. First, and most apparent, increasing barriers to entry can backfire. Starting in a world where a competitive market would result and shifting it cornered market can spike violence. To make matters worse, the Target would also waste its resources to shift $c$ in the process. Consequently, Targets do not choose such a $c$.

[13] Thus, taking the negative of is concave, leaving a strictly concave utility function overall. The specific loss functions for more effort from the target are generally unimportant in these extensions, so I save most of the details for the appendix.

[14] If $\underline{c} > c^*$, then the cost of entry is already so expensive that Group 2 will not enter the market. Increases to $c$ will not change that.

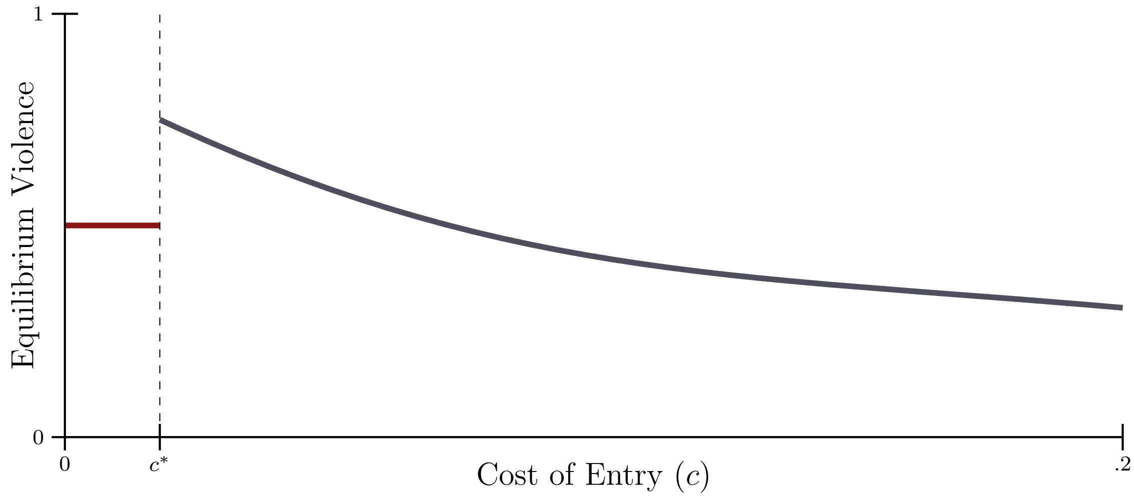


Figure 3: Equilibrium violence as a function of the cost of entry $c$, with $m_1 = m_2 = 1$. When the cost of entry is low, the competitor enters the market, and the groups produce the same amount of violence regardless of that cost of entry. When the cost of entry is high, the lead group produces enough violence to corner the market, which decreases in that cost of entry. Increasing the cost of entry may lead to an increase in violence.

Second, if a competitive market is the result of inaction and the Target chooses to increase the cost of entry, it *must* create a cornered market. This is because a shift from $\underline{c}$ to another value still below $c^*$ does not alter equilibrium violence, as in the left portion of Figure 3. Meaningful change requires moving to a cornered market, with the caveat from above that this can do more harm than good.

As the appendix details, these incentives cause the Target to play a "go big or go home" strategy in equilibrium. In particular, it calculates the optimal tradeoff between its effort to change $c$ and violence *conditional on inducing a cornered market*. If its utility for that is less than than maintaining $\underline{c}$, it chooses $\underline{c}$. If it is greater, then it picks that optimal $c$. But note from Figure 3 that this optimal $c$ must be well above $c^*$. That is because the equilibrium violence in the optimal cornered outcome must be below the amount of violence from a competitive market. Due to the discontinuous jump in violence at $c^*$, the Target must place $c$ well above $c^*$ to see any net decrease in violence. In short, the Target can never choose a half measure.

Consequently, the Lucas (1976) critique urges caution when considering the empirical implications of the results for Propositions 1. Suppose that the full empirical record reveals a monotonic decrease in violence as targets increase the barriers of entry for groups. It would be tempting to then conclude that governments seeking to reduce violence should increase the costs of entry. However, the Target never chooses a value for $c$ that increases equilibrium violence. This strategic decision therefore obscures the overall effects of $c$ in the empirical record.

## 3.2 Does Resolving Grievances Reduce Violence?

A softer approach to counterterrorism involves reducing grievances among those who would otherwise wish to lend support to an organization. Isolationists in the United States, for example, argue that abandoning foreign entanglements and deconstructing military bases in the Middle East will result in fewer attacks from radical Islamicists. France and Germany in particular cited fear of terrorism caused by increased grievances as a reason to stay out of the Iraq War (Pauly 2013, 12-13; Dettke 2009, 157-158). In strictly intrastate conflicts, separatist insurgents would seem less prone to commit violence against their home governments if granted regional autonomy. Pakistan adopted this strategy in a similar scenario against the Taliban, relinquishing control over parts of their country to the group until American pressure in 2007 made the effort worth the cost (Hoyt 2015; Rashid 2009, 385). Meanwhile, the National Strategy for Combatting Terrorism argues that democratic governance can reduce grievances by giving citizens the opportunity to address issues through their power to vote.

Nevertheless, the model demonstrates that the relationship between grievances and violence is not straightforward. Consider the following extension. Previously, the value of the pool of support equaled 1. Now, let the size of that pie equal $\pi \in [0, 1]$. The Target begins the game by selecting $\pi$. One could conceptualize selecting a greater value of $\pi$ as extracting more of a good in dispute, a foreign country having a more expansionist foreign policy (Savun and Phillips 2009), or increased repression of a minority group. Greater demands lead to a larger aggrieved population, either through direct injury or indirect economic distortions (Bueno de Mesquita 2005a). Altering $\pi$ in this manner reflects that. Let the Target's utility function equal $\beta\pi - (v_1 + v_2)$, where $\beta > 0$ is a scalar that measures the Target's value for the aggrieving policy versus violence. Meanwhile, the value of the contest for Group $i$ given Group $j$'s violence is now $\pi \left( \frac{v_i}{v_i + v_j} \right) - m_i v_i$.

As before, the appendix contains a detailed explanation of the game's equilibrium

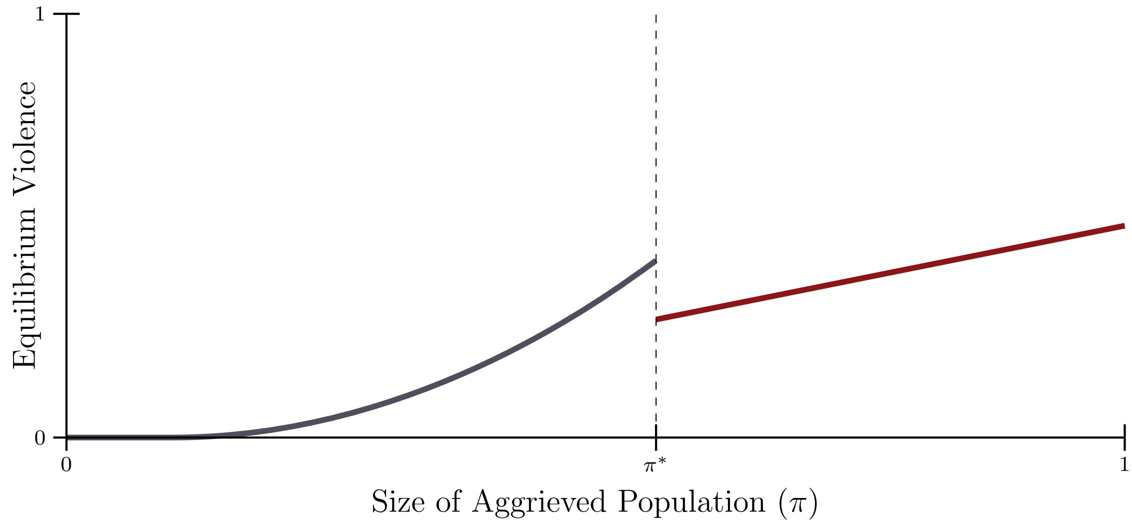


Figure 4: Equilibrium violence as a function of the percentage of supporters entering the market, with $m_1 = m_2 = 1$ and $c = .01$. Larger pools of support turn a cornered market into a competitive market, as the challenging entrant wishes to capture some market share. Counterintuitively, this means that increasing grievances can cause a decrease in overall violence.

behavior under the conditions of interest. However, the following proposition explains the key finding:

**Proposition 2.** *Decreasing grievances (i.e., decreasing $\pi$) can lead to an increase in violence.*

Figure 4 illustrates the central intuition. When the size of the pie is small, the market cannot readily support multiple groups; too few resources exist to justify Group 2's entry. Even as the pie increases to where it could support both groups, Group 1 can corner the market at a relatively low price and consequently does so. Only after there is sufficient support (i.e., $\pi > \pi^* \equiv c\left(1 - \sqrt{\frac{m_2}{m_1} - \frac{m_2^2}{4m_1^2}}\right)^{-2}$) is Group 2 willing enough to enter that Group 1 no longer wishes to overproduce to corner the market.

But this puts the Target in a familiar dilemma. If it reduces grievances among the groups' audience (that is, it moves from right to left on Figure 4), it switches a competitive market to a cornered market. And because cornering the market near the cutpoint $\pi^*$ requires Group 1 to produce more violence than the sum of their violence in a competitive market, reducing grievances can backfire.

That said, outside the discontinuity, equilibrium violence increases within the segregated parameter spaces as grievances increase. For example, in the region where Group 1 corners, the amount of violence produced decreases as $\pi$ decreases. So the groups actively respond to the size of the pie. As such, unlike with endogenous costs of entry, the Target may wish to alter grievances in a manner that does not change a cornered market to a competitive one or vice versa. However, if it wishes to change a potentially competitive market to a cornered market, half measures will not work once again. This is because the Target must place $\pi$ well below $\pi^*$ to drop the violence below where it would otherwise be just to the right of $\pi^*$.[15]

Indeed, there are many substantive examples of organizations increasing violence following concessions. Dugan and Chenoweth (2012), for instances, find that Israel experienced upticks in violence during the First Intifada and Oslo Lull following small amounts of concessions. And offering decentralization in ethnic conflicts has an inconsistent track record (Brancati 2006). Bueno de Mesquita (2005b) explains the counterintuitive relationship as the result of moderates pulling out of the organizational hierarchy; the remaining extremists correspondingly push the agenda harder than before. Alternatively, the conciliator may expect this as the cost of doing business, knowing that some individuals may wish to spoil the peaceful path forward (Stedman 1997; Kydd and Walter 2002; Findley and Young 2015).[16]

My mechanism is complementary to these and indicates that future empirical research ought to consider the counterfactual—this violence can be the result of what *would* happen in its absence, making it difficult to identify its cause. A steady decline in grievances ultimately reduces violence, but the benefits may not be immediately apparent.

## 3.3 Does Whac-a-Mole Work?

Offensive measures are a third strategy to reduce political violence (Pillar 2001, 33-34). Here, the target government actively pursues *existing* groups, attempting to reduce

[15] In Figure 4, equilibrium violence goes to 0 as $\pi$ goes to 0. This is due to the functional form that maps the Target's choice to the pool of support. One might imagine instead that some segment of the population would still want to contribute even if $\pi = 0$. Here, equilibrium violence would be bound strictly above 0 as $\pi$ goes to 0. This does not affect the main result on the discontinuity at $\pi^*$, however.

[16] A reputation mechanism (Walter 2006) can also explain a spike in violence from outside parties.

their fighters, funds, and infrastructure. Many operations fit this category: demolitions of operative housing (Benmelech, Berrebi, and Klor 2015), bombings of camps and convoys, assassinating leadership, attacks on state sponsors, seizing financial assets held abroad, and crackdowns on black market commerce, like opiates in Afghanistan and oil with ISIS. The target government may also massively broaden its intelligence net—both domestic and abroad—to assist with these tasks. Offensive measures rarely eradicate entire organizations. However, they make continued violence more expensive, as the group must alter the means of attack (Phillips 2015), operate with fewer agents, and a draw from a smaller budget to accomplish its goals.

The "Whac-A-Mole" theory of counterterrorism suggests that these results could backfire. Critics suggest that destroying one group is not helpful because another group will arise to capture the marketshare. This might be especially concerning if a strong organization replaces an otherwise enfeebled group. Yet equilibrium results indicate that Whac-A-Mole is effective.

To investigate that claim, suppose that the target begins the game by choosing $m_1 \in [\underline{m}_1, \infty)$, where $\underline{m}_1 > 0$ represents Group 1's marginal cost of violence if the Target takes no action. As always, the Target's payoff is the negative sum of the violence it suffers minus a function of the effort the Target expends to increase Group 1's marginal cost. The appendix fully solves the game in which this specific utility function equals $-(v_1 + v_2) - \delta(m_1 - \underline{m}_1)$, where $\delta$ measures how much the Target values effort versus violence; smaller values of $\delta$ reflect a greater fear of violence, as the Target finds the per-unit increase of changing the marginal cost to be less important. Like before, the key results follow as long as the effort function is convex.

The Target's decision ultimately hinges on whether to choose a marginal cost above or below a particular cutpoint. Recall that $m_1$ partially determines whether Group 1 corners the market. In terms of $c$, Group 1 cornered if $c > \left(1 - \sqrt{\frac{m_2}{m_1} - \frac{m_2^2}{4m_1^2}}\right)^2$. Solving for $m_1$ yields $m_1 < m_1^* \equiv \frac{m_2 + m_2\sqrt{2\sqrt{c} - c}}{2(1-\sqrt{c})^2}$.[17] Thus, if $m_1$ is less than $m_1^*$, Group 1 corners because the necessary overproduction of violence to exclude Group 2 appears relatively cheap. But if $m_1 > m_1^*$, cornering is too expensive, and thus Group 1 chooses

[17]Solving for $m_1$ requires using the quadratic formula, which generates two solutions. The requirements of this parameter space rule out the smaller of the two solutions, thereby generating a single relevant cutpoint.

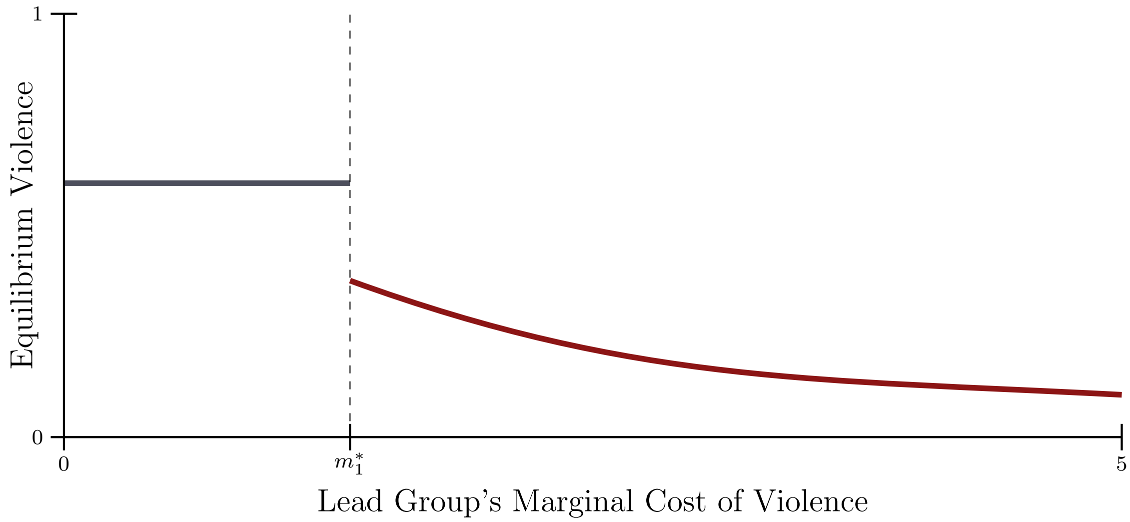


Figure 5: Equilibrium violence as a function of the lead group's marginal cost of violence, with $m_2 = 1$ and $c = .05$. Increasing the marginal cost switches the outcome from a cornered to a competitive market. Violence consequently diminishes because the lead group no longer overproduces to suppress the potential entrant.

to compete with Group 2.[18]

Because increasing the marginal cost cannot turn an otherwise competitive market into a cornered one, the following proposition states that offensive measures are mostly effective:

**Proposition 3.** *Increasing the lead group's marginal cost of violence ($m_1$) weakly decreases violence.*

Figure 5 explains the relationship. Selecting an $m_1 < m_1^*$ induces Group 1 to corner the market. This leads to a fixed level of violence because Group 1's cornering violence depends on Group 2's incentives (i.e., $c$ and $m_2$) and not Group 1's specific marginal cost. Furthermore, the level of violence is high because Group 1 must overproduce to corner the market. Pushing past $m_1^*$ leads to a discontinuous drop in violence because the outcome switches to a competitive market. Equilibrium levels of violence decline as $m_1$ increases here because Group 1 wishes to produce less in a competitive contest as its per unit cost increases.[19]

[18] If $m_1 > m_1^*$, then the Target can only select a value that leads to a competitive market.

[19] A careful reader will note that increasing $m_1$ can increase Group 2's production. However, overall

In turn, if $\underline{m}_1 < m_1^*$, the Target's decision works as follows.[20] It begins by calculating its overall payoff for maintaining a cornered market by selecting $\underline{m}_1$. The Target will not choose a value between $\underline{m}_1$ and $m_1^*$ because doing so requires costly effort but maintains an identical amount of violence as $\underline{m}_1$.[21] It then calculates the optimal tradeoff between increasing $m_1$ and the Target's own marginal cost of effort $\delta$, assuming that the market will be competitive. If the optimal $m_1$ in this calculation is greater than $m_1^*$, then Group 1 compares its utility for selecting that to its utility for keeping $\underline{m}_1$ and chooses the great option.

If the optimal $m_1$ in this calculation is less than $m_1^*$, the Target may still wish to increase $m_1^*$ to benefit from the discontinuous dropoff. It therefore compares its payoff for $\underline{m}_1$ to that of $m_1^*$ (assuming that the indifferent Group 1 pursues the competitive outcome) and picks the strategy that produces the greater payoff.[22]

Consequently, increasing the lead group's marginal cost has mostly positive effects. The lone issue is that exerting effort but choosing a value less than $m_1^*$ (instead of sticking with $\underline{m}_1$) has no net effect on violence. Thus, if the Target wishes to manipulate Group 1's marginal cost of violence in an otherwise cornered market, it must shift $m_1$ to a value that produces a competitive market. Unlike the manipulation of $c$, though, the Target might not want to go deep past $m_1^*$ because altering the market cannot increase violence. On the contrary, Group 1 might wish to go exactly to $m_1^*$ to experience the discontinuous dropoff and leave it at that.

What do the Whac-a-Mole critics miss? Their main concern is that destroying one group merely allows another organization to arise to capture the marketshare. The equilibrium results sympathize with this—increasing $m_1$ across the $m_1^*$ threshold indeed produces another group. But the introduction of another group is *exactly* what the Target wishes to induce—if no intervention would result a single group cornering

violence production equals $\frac{m_2}{4m_1^2} + \left(\frac{\sqrt{\frac{m_2}{4m_1^2}}}{m_2} - \frac{m_2}{4m_1^2}\right) = \frac{1}{2m_1}$, which is decreasing in $m_1$.

[20] If $\underline{m}_1 > m_1^*$, the the Target can only induce a competitive outcome. It therefore chooses the value of $m_1$ that optimizes the reduction in violence versus $\delta$, its marginal cost effort.

[21] Long (2014) provides empirical support for this—he finds that leadership targeting in Afghanistan and Iraq had limited effects when leadership was well-institutionalized (i.e., when $m_1$ is low) but reduced violence when leadership was poorly institutionalized (i.e., when $m_1$ is high). The nonlinear relationship in Figure 5 matches this.

[22] In fact, in any equilibrium in which the Target chooses $m_1^*$, Group 1 must produce the competitive level of violence with probability 1. This is because the Target could profitably deviate to a slightly larger value if Group 1 were to select the cornering level of violence with positive probability.

the market, the Target's manipulation *must* cause an additional group to form. This pleases the Target, as it switches the game from the cornered market outcome with an overproduction of violence to a competitive market with a comparatively smaller level of violence.

Still, Whac-a-Mole correctly cautions against the efficacy of large-scale offensive operations. Because another organization waits in the wings, there are decreasing marginal returns to enfeebling the lead group. Although the Target may wish to intervene in a competitive market, the competition nevertheless limits the effectiveness of that intervention.

Offensive operations interact with previously described manipulations in interesting ways as well. To begin, the critical value $c^*$ in Figure 7 drifts to the right as $m_1$ increases. Thus, enfeebling Group 1 means that the Target has to increase the cost of entry at a higher level than before to causes a decline in violence. Second, I have assumed that increasing $m_1$ does not increase grievances. This might not be the case for some operations, as the provocation literature highlights.[23] The bright spot here is that, per Proposition 2, the grievances facilitate switching an otherwise cornered market into a competitive one, thereby compounding the effect.

The discontinuous dropoff in violence at $m_1^*$ also has an important implication for collective counterterrorism. Throughout, I have described the Target as a single entity. Arce and Sandler (2005), however, note that transnational terrorism often strikes multiple entities. Further, they show that counterterrorism under these circumstances can be a collective action problem. Individual incentives encourage states to bolster their own defense. But this has negative externalities on other states, as those less-defended countries are more tempting terrorist targets. This leaves all states worse off than if they offensively attacked the organization because proactive measures have positive externalities. Despite the temptation to free ride, a state may wish to choose contribute to the proactive effort under the conditions of my model so that it may capture a portion of the discontinuous drop in violence.[24]

On the other hand, the model also gives an alternative explanation for a lack of

[23] See See Fromkin 1975, 962-964; Price 1977; Crenshaw 1981, 387; Berry 1987, 8-10; Laqueur 1987; Lake 2002; Bloom 2005, 107-110; Kydd and Walter 2006, 69-72; Carter 2016.

[24] Of course, this does not give a clear indication of which state should exert that effort. Nevertheless, efficient equilibria exist in volunteer's dilemmas like this, which is not the case for standard prisoner's dilemmas.

counterterrorism. When the lead group's marginal cost of violence is very low, the counterterrorist must exert a great deal of effort to reach the nonmonotonic dropoff in violence. As such, rather than contributions failing due to a collective action problem, moving the needle may simply be too expensive.

### 3.4 The Effectiveness of Defensive Measures

Finally, a target government can opt for defensive measures: hardening potential targets, stationing extra security at large gatherings, and increasing the scope of searches at airports (Pillar 2001, 37-40). The best defense may not make the target invulnerable, but it can force the organization to divert to other targets (Lum, Kennedy, and Sherley 2006), which may be less worrisome. Defense has an interesting property: it does not discriminate. Thus, in contrast of offensive measures, reinforcing security impacts all organizations, formed or unformed.

To model the consequences of defense, suppose that the game begins with the Target choosing $\gamma \in [0, 1]$, where $\gamma$ represents the portion of violence that is successful.[25] Thus, the original game is the special case where $\gamma = 1$. A value of $\gamma = \frac{2}{3}$ means only that two-thirds of the groups' effort turns into punishing violence, with the defensive measures absorbing the remaining third. The Target pays a cost that is strictly decreasing in $\gamma$ and strictly convex.[26] This means that cutting violence by larger margins is more costly, and the first measures to reduce violence are less costly than the next. From there, the game proceeds as usual except that Group $i$'s contribution to the contest is now $\gamma v_i$ instead of $v_i$.

A surprising result follows immediately from the groups' utility functions. For example, Group 1's utility equals $\frac{\gamma v_1}{\gamma v_1 + \gamma v_2} - m_1 v_1$. The $\gamma$ values immediately cancel. The same is true for Group 2's utility function, meaning that $\gamma$ *has no impact on the remaining subgame.* Group 2 still uses the same entry decision rule, and both groups choose their violence levels exactly as before. This is because the defensive measures impact both groups equally, and they choose their levels of violence where their marginal gain equals their marginal cost. The violence strategies of the original game hit this point for this extension precisely because $\gamma$ has no affect on the contest.

[25] Powell 2007 unpacks the blackbox of defense I describe here.

[26] As before, the negative of this function is strictly concave, giving the Target a strictly concave utility function overall.

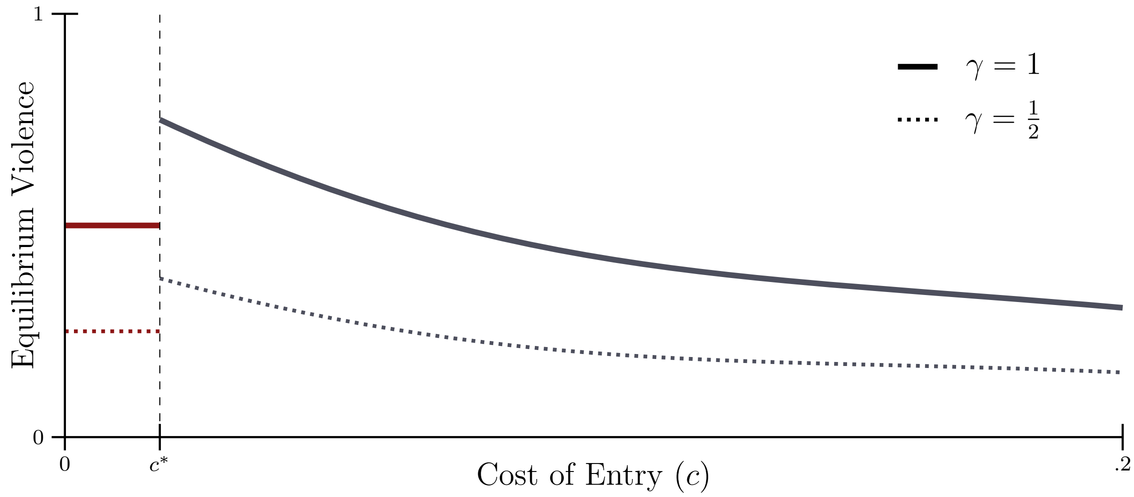


Figure 6: Equilibrium violence as a function of the lead group's cost of entry for two different levels of defensive efforts ($\gamma$). Increasing defensive measures does not impact the lead group's decision to corner the market, and violence levels are lower regardless of that choice.

In turn, the Target merely needs to optimize a straightforward tradeoff between the sum of violence realized violence $\gamma(v_1 + v_2)$ and its cost of effort. The following proposition summarizes how the Target can think about the benefits of action:

**Proposition 4.** *Increasing defensive measures (i.e., decreasing $\gamma$) strictly decreases equilibrium violence.*

Figure 6 showcases the relationship, showing how equilibrium violence changes as a function of $c$ for two levels of $\gamma$. The solid line graphs the $\gamma = 1$ case, which is a duplicate of plot in Figure 3. The dotted line tracks $\gamma = \frac{1}{2}$. The cutpoint for $c$ remains unchanged, and violence decreases by exactly half across the board. Generalizing form this, decreasing $\gamma$ further will only force equilibrium violence to decline. The figure also shows that defense is most useful in just-barely cornered markets. Violence peaks under those circumstances, making the marginal cost of defense look less onerous.

The monotonic reduction in violence here means that defensive measures have an attractive quality that the others lack: non-strategic increases to defense always pay off for the target. This ease of implementation may help explain the roughly half of U.S. counterterrorism spending funds target hardening (Rosendorff and Sandler 2004, 658).

However, the takeaway here is not that governments should exclusively rely on defense. Other strategies can work as well, and their marginal utility may be superior due to beneficial nonmontonic declines in violence. Policymakers must exert more effort to properly calibrate those strategies to ensure the benefits, though.

# 4 Conclusion

This paper explored the hidden role of potential entry into a market for political violence. Even when not facing immediate competition, hegemonic groups may still wish to produce high levels of violence to convince other political violence entrepreneurs to stay on the sidelines. Under such conditions, a potential entrant observes that it cannot capture a large share of its audience's resources and therefore does not wish to pay the costs necessary to create an organization; the existing group ultimately benefits by maintaining hegemony over the resources, though it may have to pay a high price to dominate the marketplace.

Indeed, the model revealed that total violence is often greater when the second group declines to compete. This led to a number of unexpected policy implications. Actions that make entering market look less appealing—such as increasing the barriers of entry and decreasing the pool of potential support—can counterintuitively increase violence by switching the equilibrium outcome from a competitive market to a cornered market. Target governments must therefore greatly increase the barriers of entry and greatly reduce grievances to receive any benefits. In contrast, other counterterrorism measures—offensive operations designed to destroy the infrastructure of existing groups and defensive operations designed to reduce realized violence from all groups—do not backfire in this manner. This is because they do not cause the lead group to switch from a competitive market to a cornered market, which in turn halts the jump in violence.

Overall, the model serves as a reminder that counterterrorism operations are a part of a larger strategic environment. Failure to account for optimal responses to group competition can lead to inaccurate empirical predictions and dangerous policy implications. This paper only addressed a single manner of competition—i.e., deterring market entry. The outbidding literature remains underdeveloped in thinking about how non-group actors affect competitive behavior. Future research ought to address whether similar unexpected relationships hold for alternative environments.

| Notation | Description |
|---|---|
| $v_i$ | Group $i$'s level of violence |
| $m_i$ | Group $i$'s marginal cost of violence |
| $c$ | Group 2's cost of entry |
| $\alpha$ | Target's marginal cost to increase $c$ |
| $\underline{c}$ | Minimum possible value of $c$ |
| $\beta$ | Scalar measuring Target's value for policy versus violence |
| $\pi$ | Level of grievances among the groups' audience |
| $\delta$ | Target's marginal cost to increase $m_1$ |
| $\underline{m}_1$ | Minimum possible value of $m_1$ |
| $\gamma$ | Portion of violence that Target's defense does not stop |
| $g(\gamma)$ | Function mapping level of $\gamma$ to cost of defensive measures |

Table 1: Notation of the Many Extensions

# 5 Appendix

This appendix gives thorough proofs for the main game and the propositions for all the extensions. It also solves for the Target's equilibrium actions in those extensions. For clarity, Table 1 contains a glossary of the notation of the various models.

## 5.1 Proof of the Main Model

As in the paper, I break this down sequentially.

### 5.1.1 Group 2's Violence Decision

**Lemma 1.** *Let* $v_2^* \equiv \sqrt{\frac{v_1}{m_2}} - v_1$. *In all SPE, Group 2 selects* $max\{0, v_2^*\}$.

*Proof:* Group 2's decision comes at the end of the game, so it optimizes its payoff by choosing the value of $v_2$ that maximizes:

$$\frac{v_2}{v_1 + v_2} - m_2 v_2$$

.

The first order condition is:

$$\frac{v_1 + v_2 - v_2}{(v_1 + v_2)^2} - m_2 = 0$$

$$m_2 v_2^2 + 2m_2 v_1 v_2 + v_1(m_2 v_1 - 1) = 0$$

Applying the quadratic formula to obtain the roots yields:

$$\frac{-2m_2 v_1 \pm \sqrt{4m_2^2 v_1^2 - 4m_2 v_1(m_2 v_1 - 1)}}{2m_2}$$

$$\pm\frac{\sqrt{v_1}}{\sqrt{m_2}} - v_1$$

Because $v_2 \geq 0$, $-\frac{\sqrt{v_1}}{\sqrt{m_2}} - v_1$ cannot be the solution because it is always negative. This leaves $\frac{\sqrt{v_1}}{\sqrt{m_2}} - v_1$, which itself may be negative if $v_1 > \frac{1}{m_2}$.[27] Consequently, Group 2 selects the maximum of 0 and $v_2^*$. □

#### 5.1.2 Group 2's Entry Decision

Now to the entry decision. I base the cutpoint on Group 1's chosen violence level $v_1$ because Group 1 selects $v_1$ in the next move that we need to solve for. As the following lemma shows, that level of violence determines Group 2's entry decision:

**Lemma 2.** *Let $v_1^* \equiv \frac{(1-\sqrt{c})^2}{m_2}$. Group 2 enters if $v_1 < v_1^*$, stays out if $v_1 > v_1^*$, and is indifferent between the two choices if $v_1 = v_1^*$.*

*Proof:* Consider four cases. First, suppose $v_1 \geq \frac{1}{m_2}$. By Lemma 1, Group 2 would select $v_2 = 0$ if it were to enter, giving it a payoff of 0 for the competition phase. However, to reach that point, it would have to pay a cost of $c$. Because not entering generates a payoff of 0, Group 2 must not enter.

Second, suppose $v_1 \in (v_1^*, \frac{1}{m_2})$. If Group 2 enters, it selects $v_2 = v_2^*$. Working through the contest success function, Group 2's payoff for the competition phase equals:

$$\frac{\sqrt{\frac{v_1}{m_2}} - v_1}{v_1 + \sqrt{\frac{v_1}{m_2}} - v_1} - \sqrt{\frac{v_1}{m_2}} - v_1$$

$$1 - \sqrt{m_2 v_1}$$

[27]This is a maximum because the second derivative of the objective function is $-\frac{2}{(v_1+v_2)^3}$.

Therefore, Group 2 stays out if:

$$1 - \sqrt{m_2 v_1} - c < 0$$
$$v_1 > \frac{(1-\sqrt{c})^2}{m_2}$$

This is true because the second requires this exact inequality.

Third, suppose $v_1 < v_1^*$. By analogous argument, Group 2 enters if $v_1 < \frac{(1-\sqrt{c})^2}{m_2}$. This holds here, so Group 2 enters.

Finally, suppose $v_1 = v_1^*$. By analogous argument, Group 2 is indifferent between entering and not entering, so it may mix freely between the two strategies. $\square$

#### 5.1.3 Group 1's Violence Decision

**Proposition 5.** *Suppose $m_1 > \frac{m_2}{2}$. Let $c^* \equiv \left(1 - \sqrt{\frac{m_2}{m_1} - \frac{m_2^2}{4m_1^2}}\right)^2$. In the unique SPE, Group 1 chooses $v_1 = v_1^*$ if $c \geq c^*$ and $v_1 = \frac{m_2}{4m_1^2}$ if $c < c^*$. Group 2 quits in the first case; it enters and produces $v_2^*$ in the second case.*

*Proof:* I will first prove existence by assuming that Group 2 quits if indifferent between quitting and entering. Broadly, Group 1 can select from two categories of violence: an amount at least as great as the cutpoint $c^*$ or an amount below. If Group 1 picks from the larger set, Group 2 quits. Group 1's payoff is therefore $\frac{v_1}{v_1+0} - m_1 v_1 = 1 - m_1 v_1$. Note that this amount is strictly decreasing in $v_1$—that is, any extra violence here serves no purpose to Group 1 but is costly. Consequently, no equilibria exist in which Group 1 picks a $v_1 > v_1^*$.

Now consider an amount from the smaller set. Group 2 enters, and (by Lemma 1) produces $v_2 = max\{0, \sqrt{\frac{v_1}{m_2}} - v_1\}$ in violence. Using the contest success function, provided that $\sqrt{\frac{v_1}{m_2}} - v_1 > 0$, Group 1's payoff equals:

$$\frac{v_1}{v_1 + \sqrt{\frac{v_1}{m_2}} - v_1} - m_1 v_1$$
$$\sqrt{m_2 v_1} - m_1 v_1$$

The first order condition of that objective function is:

$$\frac{\sqrt{m_2}}{2\sqrt{v_1}} - m_1 = 0$$

$$v_1 = \frac{m_2}{4m_1^2}$$

This a maximum because the second derivative of the objective function is $-\frac{m_2^{\frac{1}{2}} v_1^{-\frac{3}{2}}}{4}$, which is negative. Further, note that $v_1 = \frac{m_2}{4m_1^2}$ implies that $\sqrt{\frac{v_1}{m_2}} - v_1 > 0$ because the proposition assumes $m_1 > \frac{m_2}{2}$. This in turn means that Group 1 maximizes its payoff with Group 2 still playing a violence strategy on the interior.

Now compare Group 1's utility for the minimum amount of violence necessary to drive Group 2 out to Group 1's optimal competitive quantity. If Group 1 drives Group 2 out at the lowest price, it earns $1 - m_1 v_1^*$. Meanwhile, choosing the optimal competitive amount produces a payoff of $\frac{\frac{m_2}{4m_1^2}}{\frac{m_2}{4m_1^2} + v_2^*} - m_1 \frac{m_2}{4m_1^2} = \frac{m_2}{4m_1}$. Thus, Group 1 induces entry if:

$$\frac{m_2}{4m_1} > 1 - m_1 \frac{(1-\sqrt{c})^2}{m_2}$$

$$c > \left(1 - \sqrt{\frac{m_2}{m_1} - \frac{m_2^2}{4m_1^2}}\right)^2$$

This is the cutpoint given in the proposition.[28]

By analogous argument, Group 1 chooses $v_1^*$ to corner the market if $c < \left(1 - \sqrt{\frac{m_2}{m_1} - \frac{m_2^2}{4m_1^2}}\right)^2$. The argument also shows that it is an equilibrium to corner the market if $c = \left(1 - \sqrt{\frac{m_2}{m_1} - \frac{m_2^2}{4m_1^2}}\right)^2$.

I will now prove uniqueness.[29] Consider the same two cases as before, beginning with $v_1^* \leq \frac{m_2}{4m_1^2}$. The above proof showed that Group 1 cannot choose an amount other

[28]Note that the value inside of the radical is positive if $m_1 > \frac{m_2}{4}$. This must be true because the parameter space has the more stringent requirement that $m_1 > \frac{m_2}{2}$. Thus, although the causes of terrorism are complex, the cutpoint is not. One might also be concerned that the optimal competitive level of violence is greater than the minimum necessary to exclude Group 2. (This could be the case if $c$ is high.) However, the inequality still produces the correct result because the left hand side represents a value that gives less than the whole prize and at a greater cost than the right hand side. Thus, the inequality would tell us that Group 1 would pick the quantity to keep out Group 2.

[29]The structure of these uniqueness proofs is similar to the uniqueness proof the ultimatum game. The receiver (Group 2) is indifferent when the proposer (Group 1) selects a particular value. But because rejecting (entering the market) leads to a dropoff in utility for the proposer (Group 1), and because the proposer (Group 1) could deviate to an offer (level of violence) slightly greater to break indifference, equilibrium constraints guarantee a unique equilibrium.

than $v_1^*$ in equilibrium. It remains to be seen whether equilibria exist in which Group 1 chooses $v_1^*$ and Group 2 enters with positive probability. (This is rational for Group 2 because it is indifferent between entering and quitting when $v_1 = v_1^*$.) So suppose that Group 2 enters with probability $\sigma \in (0, 1]$. Group 1's utility for selecting $v_1^*$ now equals:

$$\sigma \left[ \frac{\frac{(1-\sqrt{c})^2}{m_2}}{\frac{(1-\sqrt{c})^2}{m_2} + v_2^*} - m_1 \left( \frac{(1-\sqrt{c})^2}{m_2} \right) \right] + (1-\sigma) \left[ 1 - m_1 \left( \frac{(1-\sqrt{c})^2}{m_2} \right) \right]$$

Note that because $\frac{\frac{(1-\sqrt{c})^2}{m_2}}{\frac{(1-\sqrt{c})^2}{m_2} + v_2^*} < 1$, this is less than Group 1's utility when Group 2 quits with probability 1. Also note that Group 1's utility is continuous (and decreasing) as it selects a level of violence greater than $\frac{(1-\sqrt{c})^2}{m_2}$. Thus, there exists an $\epsilon > 0$ such that

$$\sigma \left[ \frac{\frac{(1-\sqrt{c})^2}{m_2}}{\frac{(1-\sqrt{c})^2}{m_2} + v_2^*} - m_1 \left( \frac{(1-\sqrt{c})^2}{m_2} \right) \right] + (1-\sigma) \left[ 1 - m_1 \left( \frac{(1-\sqrt{c})^2}{m_2} \right) \right]$$
$$< 1 - m_1 \left( \frac{(1-\sqrt{c})^2}{m_2} + \epsilon \right)$$

holds. This means that if Group 2 enters with positive probability, Group 1 can profitably deviate. In turn, the equilibrium in Proposition 5 is unique when $v_1^* \leq \frac{m_2}{4m_1^2}$.

Now consider the second case, in which $v_1^* > \frac{m_2}{4m_1^2}$. If Group 1 selects $\frac{m_2}{4m_1^2}$, the existence proof showed that Group 2 has a unique best response to enter and produce $v_2^*$. If Group 1 selects $\frac{(1-\sqrt{c})^2}{m_2}$ instead, Group 2 is indifferent between entering and quitting. But if Group 2 enters with positive probability, the same uniqueness argument as above applies—i.e., Group 1 has a profitable deviation to some $v_1 = \frac{(1-\sqrt{c})^2}{m_2} + \epsilon$. Therefore, the strategies presented in Proposition 5 are unique. □

**Proposition 6.** *Suppose $m_1 \leq \frac{m_2}{2}$. In the unique SPE, Group 1 chooses $v_1 = v_1^*$ and Group 2 quits.*

*Proof:* By Lemma 2, if Group 1 selects $v_1 > v_1^*$, Group 2 quits. No equilibria exist for $v_1 > v_1^*$ because Group 1 can profitably deviating to a slightly smaller level of violence still greater than $v_1^*$.

If Group 1 selects $v_1 \in (0, v_1^*)$, Group 2 enters and produces $v_2^*$ violence. Group 1's utility equals $\frac{v_1}{v_1+v_2^*} - m_1 v_1$. The first order condition from Proposition 1 demonstrates that this function is strictly increasing until $v_1 = \frac{m_2}{4m_1^2}$. Note that for this proposition's parameter space ($m_1 \leq \frac{m_2}{2}$), $\frac{m_2}{4m_1^2}$ is greater than $v_1^*$. Thus, Group 1's utility is strictly increasing on the interval $v_1 \in (0, v_1^*)$. In turn, no equilibrium can exist on that range because Group 1 could profitably deviate to a level of violence slightly greater while still below $v_1^*$.

The only case left to check is when Group 1 produces exactly $v_1^*$. Group 2 is indifferent between entering and quitting. The proof for Proposition 5 showed that Group 1 receives strictly more when Group 2 quits under these circumstances. As such, this is an equilibrium. Furthermore, the proof for Proposition 5 also showed that Group 1 could profitably deviate to a slightly greater level of violence if Group 2 were to enter with positive probability when indifferent. The aforementioned equilibrium is therefore unique. □

## 5.2 Endogenous Fixed Costs of Entry

**Proposition 7.** *Let* $c^{**} \equiv \frac{1}{(\alpha m_2 - 1)^2}$. *If* $m_1 < \frac{m_2}{2}$, *the Target chooses* $max\{\underline{c}, c^{**}\}$. *If* $m_1 > \frac{m_2}{2}$, *the following four cases describe the Target's unique equilibrium action:*

1. *If* $\underline{c} > c^*$ *and* $\underline{c} > c^{**}$, *the Target chooses* $\underline{c}$.
2. *If* $\underline{c} > c^*$ *and* $\underline{c} < c^{**}$, *the Target chooses* $c^{**}$.
3. *If* $\underline{c} < c^*$ *and* $c^{**} > c^*$, *the Target chooses* $\underline{c}$.
4. *If* $\underline{c} < c^*$ *and* $c^{**} < c^*$, *the Target chooses* $c^{**}$ *if* $-\frac{(1-\sqrt{c^{**}})^2}{m_2} - \alpha(c^{**} - \underline{c}) > -\frac{1}{2m_1}$ *and* $\underline{c}$ *if* $-\frac{(1-\sqrt{c^{**}})^2}{m_2} - \alpha(c^{**} - \underline{c}) < -\frac{1}{2m_1}$.

*Proof*: Recall that the Target's objective function is $-(v_1 + v_2) - \alpha c$. From Propositions 5 and 6, Group 1 produces $v_1 = \frac{m_2}{4m_1^2}$ if $c < c^*$. Group 2 enters and produces $v_2^*$, generating a total of $\frac{1}{2m_1}$ in violence. If $c \geq c^*$, Group 1 produces $v_1^* = \frac{(1-\sqrt{c})^2}{m_2}$ violence to force out Group 2.

Note that increasing the cost of entry does not affect equilibrium levels of violence if the chosen cost is low enough to fall in the competitive case. If the Target wishes to push the cost of entry into the greater case, it must maximize:

$$-\frac{(1-\sqrt{c})^2}{m_2} - \alpha(c - \underline{c})$$

The first order condition is:

$$-\frac{\left(1-\frac{1}{\sqrt{c}}\right)}{m_2} - \alpha = 0$$

$$c = c^{**} = \frac{1}{(\alpha m_2 - 1)^2}$$

Recall that Proposition 6 showed that there is no cost for which Group 2 enters under its parameters. Thus, Group 2 does not enter regardless of the Target's decision. In turn, the Target only has to worry about the above first order condition. It therefore chooses the solution to the optimization problem $c^{**}$ if it is greater than the minimum cost $\underline{c}$. If $c^{**} < \underline{c}$, then the first order condition showed that any further cost increases lead to a decrease in utility, so the Target chooses $\underline{c}$.

Now to the cases. First, suppose $\underline{c} > c^*$ and $\underline{c} > c^{**}$. This means that the minimum cost of entry is greater than both the highest possible cost for a competitive equilibrium and the Target's optimal cost in the cornering equilibrium. Since the first order condition shows that increasing $c$ any further leads to a lower utility, the Target optimally picks $\underline{c}$.

Second, suppose $\underline{c} > c^*$ and $\underline{c} < c^{**}$. The minimum cost of entry remains greater that the highest possible cost for a competitive equilibrium but is less than the Target's optimal cost in the cornering equilibrium. The first order condition showed that selecting $c^{**}$ is optimal here.

Third, suppose $\underline{c} < c^*$ and $c^{**} > c^*$. The minimum cost of entry is less than the highest possible cost for a competitive equilibrium but is now greater than the Target's optimal cost in the cornering equilibrium. The optimal cost for maintaining the competitive equilibrium remans $\underline{c}$ because anything greater needlessly exerts effort. Its payoff for choosing that amount equals $-\frac{1}{2m_1}$. The optimal cost to shift into the cornering equilibrium switches to $c^*$ because the first order condition showed that the Target's payoff is decreasing going away from $c^{**}$. Its payoff for choosing that amount equals $-\frac{(1-\sqrt{c^*})^2}{m_2} - \alpha(c^* - \underline{c})$. The Target therefore chooses $\underline{c}$ if

$$-\frac{1}{2m_1} > -\frac{(1-\sqrt{c^*})^2}{m_2} - \alpha(c^* - \underline{c})$$

I can show this holds by instead demonstrating

$$-\frac{1}{2m_1} > -\frac{(1-\sqrt{c^*})^2}{m_2}$$

Substitution and substantial algebraic manipulation yields $m_1 > \frac{1}{2m_1}$, which holds. So the Target chooses $\underline{c}$.

Finally, suppose $\underline{c} < c^*$ and $c^{**} < c^*$. Now the minimum cost of entry is less than the highest possible cost for a competitive equilibrium and is also less than the Target's optimal cost in the cornering equilibrium. Thus, the Target can manipulate whether Group 1 induces Group 2 to enter. If the Target wishes to deter entry, the first order condition showed the optimal cost for doing so is $c^{**}$. This generates a payoff of $-\frac{(1-\sqrt{c^{**}})^2}{m_2} - \alpha(c^{**} - \underline{c})$. Alternatively, the Target can maintain the competitive equilibrium; the optimal cost to do this is $\underline{c}$ because any additional cost lowers the Target's payoff without manipulating the remainder of the game. The Target earns $-\frac{1}{2m_1}$ for this. Thus, it prefers selecting $c^{**}$ if $-\frac{(1-\sqrt{c^{**}})^2}{m_2} - \alpha(c^{**} - \underline{c}) > -\frac{1}{2m_1}$ and prefers selecting $\underline{c}$ if $-\frac{(1-\sqrt{c^{**}})^2}{m_2} - \alpha(c^{**} - \underline{c}) < -\frac{1}{2m_1}$. □

## 5.3 Endogenous Grievances

I begin by solving the two player interaction with Group 1 and Group 2. The proof strategy follows the strategy of the original game, as it is a special case of this version with $\pi = 1$.

**Group 2's Violence Decision.** At the end of the game, if Group 2 has entered, its objective function is:

$$\frac{v_2}{v_1 + v_2}(\pi) - m_2 v_2$$

The first order condition is:

$$\frac{v_1}{(v_1 + v_2)^2}(\pi) - m_2 = 0$$

$$v_2 \equiv \sqrt{\frac{\pi v_1}{m_2}} - v_1$$

This is negative if $v_1 > \frac{\pi}{m_2}$, so Group 2 chooses $max\{\sqrt{\frac{\pi v_1}{m_2}} - v_1, 0\}$.[30]

**Group 2's Entry Decision.** If $v_1 \geq \frac{\pi}{m_2}$, Group 2 produces 0 for the contest and thus receives 0 as its payoff for the contest. Since entering costs $c$, Group 2 will not enter if $v_1 > \frac{\pi}{m_2}$.

If $v_1 < \frac{\pi}{m_2}$, Group 2 produces $\sqrt{\frac{\pi v_1}{m_2}} - v_1$. Routing this through its utility function for the contest, entering produces $\frac{\sqrt{\frac{\pi v_1}{m_2}}-v_1}{v_1+\sqrt{\frac{\pi v_1}{m_2}}-v_1}(\pi) - m_2\left(\sqrt{\frac{\pi v_1}{m_2}} - v_1\right)$ at cost $c$. Quitting generates 0. Group 2 therefore enters if:

$$\frac{\sqrt{\frac{\pi v_1}{m_2}} - v_1}{v_1 + \sqrt{\frac{\pi v_1}{m_2}} - v_1}(\pi) - m_2\left(\sqrt{\frac{\pi v_1}{m_2}} - v_1\right) - c > 0$$

$$c < (\sqrt{\pi} - \sqrt{m_2 v_1})^2$$

Analogously, Group 2 quits if $c > (\sqrt{\pi} - \sqrt{m_2 v_1})^2$ and is indifferent when $c = (\sqrt{\pi} - \sqrt{m_2 v_1})^2$.

**Group 1's Violence Decision.** The minimum cost necessary to drive Group 2 is $c = (\sqrt{\pi} - \sqrt{m_2 v_1})^2$.[31] Solving for $v_1$ yields $\frac{(\sqrt{\pi}-\sqrt{c})^2}{m_2}$. Thus, Group 2 enters if Group 1 selects $v_1 < \frac{(\sqrt{\pi}-\sqrt{c})^2}{m_2}$ and stays out if $v_1 \geq \frac{(\sqrt{\pi}-\sqrt{c})^2}{m_2}$.[32] In turn, Group 1's optimal level of violence to exclude Group 2 equals $\frac{(\sqrt{\pi}-\sqrt{c})^2}{m_2}$

If Group 1 wishes to induce Group 2's entry, its objective function is:

$$\frac{v_1}{v_1 + \sqrt{\frac{\pi v_1}{m_2}} - v_1}(\pi) - m_1 v_1$$

The first order condition is:

[30]This is maximum because the second derivative is $-\frac{2\pi}{(v_1+v_2)^3}$.

[31]I assume that Group 2 stays out when indifferent, but this must be true in equilibrium as before.

[32]The out decision covers the case where $v_1 > \frac{\pi}{m_2}$, which would otherwise result in Group 2 producing 0 violence.

$$\frac{\sqrt{\pi m_2}}{2\sqrt{v_1}} - m_1 = 0$$

$$v_1 = \frac{\pi m_2}{4m_1^2}$$

Selecting this value induces Group 2 to choose a strictly positive quantity of violence if $m_1 > \frac{m_2}{2}$. If $m_1 < \frac{m_2}{2}$ instead, then Group 1 selects $\frac{(\sqrt{\pi}-\sqrt{c})^2}{m_2}$. (This is analogous to Proposition 6.)

Now for the $m_1 > \frac{m_2}{2}$ case. If Group 1 excludes Group 2, it earns $\pi - (m_1)\frac{(\sqrt{\pi}-\sqrt{c})^2}{m_2}$. If it produces the optimal competitive amount and Group 2 enters, Group 1's utility is the above objective function with $v_1 = \frac{\pi m_2}{4m_1^2}$. It therefore excludes Group 2 if:

$$\pi - (m_1)\frac{(\sqrt{\pi} - \sqrt{c})^2}{m_2} > \frac{\frac{\pi m_2}{4m_1^2}}{\frac{\pi m_2}{4m_1^2} + \sqrt{\frac{(\pi)\frac{\pi m_2}{4m_1^2}}{m_2}} - \frac{\pi m_2}{4m_1^2}}(\pi) - m_1\frac{\pi m_2}{4m_1^2}$$

$$\pi < \pi^* \equiv \frac{c}{\left(1 - \sqrt{\frac{m_2}{m_1} - \frac{m_2^2}{4m_1^2}}\right)^2}$$

By analogous argument, Group 1 induces entry if $\pi > \pi^*$ and is indifferent when $\pi = \pi^*$.[33]

**Violence at the Cutpoint.** As with the other extensions, the quantity of violence near the cutpoint drives the Target's decision. As $\pi$ approaches $\pi^*$ from the left, Group 1 corners and produces $\frac{(\sqrt{\pi}-\sqrt{c})^2}{m_2}$ in violence. As $\pi$ approaches $\pi^*$ from the right, the parties compete; Group 1 produces $\frac{\pi m_2}{4m_1^2}$ and Group 2 produces $\sqrt{\frac{\pi\left(\frac{\pi m_2}{4m_1^2}\right)}{m_2}} - \frac{\pi m_2}{4m_1^2}$. Substituting $\pi = \pi^*$, violence in the cornering case is greater than the sum of violence in the competitive case if:

[33] As with the baseline model, this covers the case where the optimal amount from the first order condition exceeds the minimum necessary amount to force out Group 2; if this were the case, the utility on the right hand side would produce less than $\pi$ from the contest and at a greater cost than the left hand side, even though the left hand side gives the full $\pi$ to Group 1.

$$\frac{\left(\sqrt{\frac{c}{\left(1-\sqrt{\frac{m_2}{m_1}-\frac{m_2^2}{4m_1^2}}\right)^2}}-\sqrt{c}\right)^2}{m_2} > \frac{\left(\frac{c}{\left(1-\sqrt{\frac{m_2}{m_1}-\frac{m_2^2}{4m_1^2}}\right)^2}\right)m_2}{4m_1^2}+$$

$$\sqrt{\frac{\left(\frac{c}{\left(1-\sqrt{\frac{m_2}{m_1}-\frac{m_2^2}{4m_1^2}}\right)^2}\right)\left(\frac{\left(\frac{c}{\left(1-\sqrt{\frac{m_2}{m_1}-\frac{m_2^2}{4m_1^2}}\right)^2}\right)m_2}{4m_1^2}\right)}{m_2}}-\frac{\left(\frac{c}{\left(1-\sqrt{\frac{m_2}{m_1}-\frac{m_2^2}{4m_1^2}}\right)^2}\right)m_2}{4m_1^2}$$

This reduces to $m_1 > \frac{m_2}{2}$. This must be true at the cutpoint, otherwise Group 1 would have a strict preference to corner.

**The Target's Grievance Decision.** We are now ready for the main proposition of this extension:

**Proposition 8.** *Let $\pi^{**} = \frac{c}{(1-\beta m_2)^2}$. The following four cases describe the Target's unique equilibrium action:*

1. *If $\beta > \frac{1}{2m_1}$ and $\pi^{**} < \pi^*$, the Target chooses $\pi = 1$ if $\beta - \frac{1}{2m_1} > \beta\pi^{**} - \frac{(\sqrt{\pi^{**}}-\sqrt{c})^2}{m_2}$ and $\pi^{**}$ if $\beta - \frac{1}{2m_1} < \beta\pi^{**} - \frac{(\sqrt{\pi^{**}}-\sqrt{c})^2}{m_2}$.*
2. *If $\beta > \frac{1}{2m_1}$ and $\pi^{**} > \pi^*$, the Target chooses $\pi = 1$.*
3. *If $\beta < \frac{1}{2m_1}$ and $\pi^{**} < \pi^*$, the Target chooses $\pi = \pi^*$ if $\beta\pi^* - \frac{\pi^*}{2m_1} > \beta\pi^{**} - \frac{(\sqrt{\pi^{**}}-\sqrt{c})^2}{m_2}$ and $\pi = \pi^{**}$ if $\beta\pi^* - \frac{\pi^*}{2m_1} < \beta\pi^{**} - \frac{(\sqrt{\pi^{**}}-\sqrt{c})^2}{m_2}$.*
4. *If $\beta < \frac{1}{2m_1}$ and $\pi^{**} > \pi^*$, the Target chooses $\pi = \pi^*$.*

*Proof:* The Target's utility function for a chosen $\pi$ depends on whether it is greater than or less than $\pi^*$. For values greater than $\pi^*$, that function is $\beta\pi - \frac{\pi}{2m_1}$. The first derivative of this is:

$$\beta - \frac{1}{2m_1}$$

Therefore, the Target maximizes its utility on this region at $\pi = 1$ if $\beta > \frac{1}{2m_1}$ and at $\pi = \pi^*$ if $\beta < \frac{1}{2m_1}$.[34]

For values less than $\pi^*$, the Target's utility function is $\beta\pi - \frac{(\sqrt{\pi}-\sqrt{c})^2}{m_2}$. The first order condition of this is:

$$\beta - \frac{1-\sqrt{\frac{c}{\pi}}}{m_2} = 0$$

$$\pi = \pi^{**} = \frac{c}{(1-\beta m_2)^2}$$

Therefore, the Target maximizes its utility on this region at $\pi^{**}$ if $\pi^{**} < \pi^*$ and at $\pi^*$ if $\pi^{**} > \pi^*$.

Now consider the four cases. If $\beta > \frac{1}{2m_1}$ and $\pi^{**} < \pi^*$, the Target most prefers selecting $\pi = 1$ in the higher region and $\pi^{**}$ in the lower region. It therefore chooses whichever produces the greater expected utility. That is, the Target chooses $\pi = 1$ if $\beta - \frac{1}{2m_1} > \beta\pi^{**} - \frac{(\sqrt{\pi^{**}}-\sqrt{c})^2}{m_2}$ and $\pi^{**}$ if $\beta - \frac{1}{2m_1} < \beta\pi^{**} - \frac{(\sqrt{\pi^{**}}-\sqrt{c})^2}{m_2}$.

If $\beta > \frac{1}{2m_1}$ and $\pi^{**} < \pi^*$, the Target still most prefers selecting $\pi = 1$ in the higher region but now prefers $\pi^*$ in the lower region. Notice, however, that the Target's expected utility for $\pi^*$ for the cornered market is less than its expected utility at $\pi^*$ in the competitive market. Furthermore, the Target's utility is strictly increasing in $\pi$ above $\pi^*$. This implies that the utility for $\pi = 1$ must be greater than the utility for $\pi^*$ in a cornered market. Therefore, the Target must select $\pi = 1$.

If $\beta < \frac{1}{2m_1}$ and $\pi^{**} < \pi^*$, the Target now most prefers selecting $\pi^*$ from the higher region and $\pi^{**}$ from the lower region. For simplicity, assume that Group 1 chooses a competitive market when indifferent between cornering and competing.[35] Then the Target chooses whichever of the two produces the greater expected utility. That is, the Target chooses $\pi = \pi^*$ if $\beta\pi^* - \frac{\pi^*}{2m_1} > \beta\pi^{**} - \frac{(\sqrt{\pi^{**}}-\sqrt{c})^2}{m_2}$ and $\pi = \pi^{**}$ if $\beta\pi^* - \frac{\pi^*}{2m_1} < \beta\pi^{**} - \frac{(\sqrt{\pi^{**}}-\sqrt{c})^2}{m_2}$.

Finally, if $\beta < \frac{1}{2m_1}$ and $\pi^{**} > \pi^*$, the Target still most prefers selecting $\pi^*$ from

[34] There is no interior solution because the mapping of $\pi$ to violence is affine in this region.

[35] As before, and for the same reasons, this assumption is actually required for equilibria to exist. That is, if Group 1 cornered with positive probability when $\pi = \pi^*$, the Target could profitably deviate to some slightly larger amount.

the higher region and now also prefers selecting $\pi^*$ from the lower region. It therefore must select $\pi^*$. Following this, as an equilibrium condition, Group 1 must select a competitive market.[36]

## 5.4 Endogenous Marginal Costs of Violence for the Lead Group

**Proposition 9.** *Let* $m_1^* \equiv \frac{m_2+m_2\sqrt{2\sqrt{c}-c}}{2(1-\sqrt{c})^2}$ *and* $m_1^{**} \equiv \frac{1}{\sqrt{2\delta}}$. *The following five cases describe the Target's unique equilibrium action:*

1. *If* $\underline{m}_1 > m_1^*$ *and* $\underline{m}_1 > m_1^{**}$, *the Target chooses* $\underline{m}_1$.

2. *If* $\underline{m}_1 > m_1^*$ *and* $\underline{m}_1 < m_1^{**}$, *the Target chooses* $m_1^{**}$.

3. *If* $\underline{m}_1 < m_1^*$ *and* $\underline{m}_1 > m_1^{**}$, *the Target chooses* $\underline{m}_1$ *if* $-\frac{(1-\sqrt{c})^2}{m_2} > -\frac{1}{2m_1^*} - \delta(m_1^* - \underline{m}_1)$ *and* $m_1^*$ *if* $-\frac{(1-\sqrt{c})^2}{m_2} < -\frac{1}{2m_1^*} - \delta(m_1^* - \underline{m}_1)$.

4. *If* $\underline{m}_1 < m_1^*$, $\underline{m}_1 < m_1^{**}$, *and* $m_1^{**} < m_1^*$, *the Target chooses according to the rule in (3).*

5. *If* $\underline{m}_1 < m_1^*$, $\underline{m}_1 < m_1^{**}$, *and* $m_1^{**} > m_1^*$, *the Target chooses* $\underline{m}_1$ *if* $-\frac{(1-\sqrt{c})^2}{m_2} < -\frac{1}{2m_1^{**}} - \delta(m_1^{**} - \underline{m}_1)$ *and* $m_1^{**}$ *if* $-\frac{(1-\sqrt{c})^2}{m_2} > -\frac{1}{2m_1^{**}} - \delta(m_1^{**} - \underline{m}_1)$.

*Proof:* To begin, we must restate Proposition 5 in terms of $m_1$. Recall from the proof that the Group 1 prefers inducing entry if:

$$\frac{m_2}{4m_1} > 1 - m_1\frac{(1-\sqrt{c})^2}{m_2}$$

Analogously, Group 1 prefers cornering the market if the inequality is flipped and is indifferent when those two values are equal.

We can rewrite this inequality as:

$$4(1-\sqrt{c})^2 m_1^2 - 4m_2 m_1 + m_2^2 > 0$$

Applying the quadratic formula yields the following roots:

[36] Again, this is because the Target could profitably deviate to a slightly greater amount otherwise.

$$\frac{m_2 \pm \sqrt{(4m_2)^2 - 4[4(1-\sqrt{c})^2](m_2^2)}}{2[4(1-\sqrt{c})]}$$

$$\frac{m_2 \pm m_2\sqrt{2\sqrt{c}-c}}{2(1-\sqrt{c})^2}$$

Because the coefficient on the lead term of the polynomial is negative, Group 1 induces entry if $m_1 < \frac{m_2 - m_2\sqrt{2\sqrt{c}-c}}{2(1-\sqrt{c})^2}$. However, recall that Proposition 5 only applies if $m_1 > \frac{m_2}{2}$. The smaller root is therefore irrelevant if:

$$\frac{m_2 - m_2\sqrt{2\sqrt{c}-c}}{2(1-\sqrt{c})^2} < \frac{m_2}{2}$$

$$c < 1$$

This is true. Thus, Group 1 corners the market up until $m_1^*$ and induces competition afterward. As such, the Target's choice is whether to keep $m_1$ small and allow for Group 1 to corner or force Group 1 to compete with Group 2 by raising $m_1^*$. In the cornered market case, equilibrium violence equals $\frac{(1-\sqrt{c})^2}{m_2}$. In the competitive market case, equilibrium violence equals $\frac{1}{2m_1}$. In this second case, the Target's utility equals $-\frac{1}{2m_1} - \delta(m_1 - \underline{m}_1)$. Thus, the first order condition is:

$$\frac{1}{2m_1^2} - \delta = 0$$

$$m_1^{**} = \frac{1}{\sqrt{2\delta}}$$

Without constraints, this is the best marginal cost the Target can create if Group 1 induces competition.[37]

Now to the cases. First, suppose $\underline{m}_1 > m_1^*$ and $\underline{m}_1 > m_1^{**}$. This means that the minimum marginal cost is greater than the highest possible cost for the cornered market outcome and also higher than the Target's optimal marginal cost within the competitive outcome. Consequently, the Target can only induce a competitive equilibrium, and any additional marginal costs only push the Target further from its optimal $m_1$. In turn, the Target chooses $\underline{m}_1$.

Second, suppose $\underline{m}_1 > m_1^*$ and $\underline{m}_1 > m_1^{**}$. The minimum marginal cost is still

[37]It is a maximum because the second order condition is $-\frac{1}{m_1^3}$.

greater than the highest possible cost for the cornered market outcome but is now less than the Target's optimal marginal cost within the competitive outcome. The first order condition above showed that $m_1^{**}$ is the Target's optimal choice here.

Third, suppose $\underline{m}_1 < m_1^*$ and $\underline{m}_1 > m_1^{**}$. Now the minimum marginal cost is less than the highest possible cost for the cornered market outcome but is greater than the optimal marginal cost for a competitive outcome. Increasing $m_1$ within the cornered range cannot be optimal because it increases the Target's effort without changing the total violence. Increasing past $m_1^*$ cannot be optimal either because the Target's utility for a competitive equilibrium is strictly decreasing past $m_1^{**}$. Therefore, the possible equilibrium amounts are $\underline{m}_1$ and $m_1^*$.

Although $m_1^*$ is past the Target's optimal competitive market marginal cost, it may still yet be optimal. This is because there is a discontinuous dropoff between violence in the cornered market case and in the competitive case if:

$$\frac{(1-\sqrt{c})^2}{m_2} > \frac{1}{2m_1^*}$$

$$\sqrt{c} < 2$$

This is true and is what guarantees that increasing $m_1$ leads to a decrease in violence even though it may switch the outcome from a cornered to a competitive market. Since Group 1 is indifferent between the cornering the market and the competitive outcome at $m_1^*$, assume that it opts for the competitive market with probability 1 in this case.[38] In turn, the Target prefers keeping the marginal cost at $\underline{m}_1$ if

$$-\frac{(1-\sqrt{c})^2}{m_2} > -\frac{1}{2m_1^*} - \delta(m_1^* - \underline{m}_1)$$

Analogously, the Target shifts the marginal cost to $m_1^*$ if the inequality is reversed.

Fourth, suppose $\underline{m}_1 < m_1^*$, $\underline{m}_1 < m_1^{**}$, and $m_1^{**} < m_1^*$. The minimum marginal cost remains less than the highest possible cost for the cornered market, but it is now less than the optimal marginal cost for a competitive outcome. Further, the optimal marginal cost for a competitive outcome falls in the region where Group 1 would corner the market. The Target again has no incentive to change the marginal cost within the

[38] In fact, Group 1 must induce entry when indifferent in equilibrium. This is for the familiar reason: if Group 1 were to choose the monopoly amount of violence, the Target could profitably deviate to $m_1^* + \epsilon$. This same uniqueness logic prevails for the other cases.

cornered market region. If it were to push to the competitive outcome, it would not want to increase the marginal cost past the cutpoint $m_1^*$ because this is already beyond the optimal competitive marginal cost. Thus, the optimization problem is identical to the third case.

Finally, suppose $\underline{m}_1 < m_1^*$, $\underline{m}_1 < m_1^{**}$, and $m_1^{**} > m_1^*$. This is the same as the fourth, except now the optimal competitive marginal cost is greater than the cutpoint that separates the outcomes. Thus, if the Target wished to increase the marginal cost to induce a competitive market, it would select $m_1^{**}$. Keeping it at $\underline{m}_1$ is optimal if

$$-\frac{(1-\sqrt{c})^2}{m_2} < -\frac{1}{2m_1^{**}} - \delta(m_1^{**} - \underline{m}_1)$$

Analogously, the Target shifts the marginal cost to $m_1^{**}$ if the inequality is reversed.

□

## 5.5 Endogenous Defense

**Proposition 10.** *Let $V^* = v_1 + v_2$, where $v_1$ and $v_2$ equal the equilibrium violence levels given by Propositions 5 and 6. The following three cases describe the Target's unique equilibrium action:*

1. *If $g'(\gamma) = -V^*$ for some $\gamma \in [0,1]$, choose the $\gamma$ that solves the equation. (The solution is unique.)*

2. *If $g'(\gamma) > -V^*$ for all $\gamma \in [0,1]$, choose $\gamma = 1$.*

3. *If $g'(\gamma) < -V^*$ for all $\gamma \in [0,1]$, choose $\gamma = 0$.*

*If $\gamma > 0$, Groups 1 and 2 then play strategies according to Propositions 5 and 6, which are not a function of $\gamma$. If $\gamma = 0$, the groups produce no violence.*

The main paper showed that the decisions from Groups 1 and 2 are not a function of $\gamma$. (The value of $\gamma$ cancels out in the contest function.) The only exception is when $\gamma = 0$, as this creates a divide by 0 issue. The utilities for each group depend on how one defines the results of the contest when all effort equals 0, but this is inconsequential for the Target's decision.

So consider the Target's objective function, $-\gamma(v_1 + v_2) - g(\gamma)$, where $g(1) = 0$, $g'(\gamma) > 0$ and $g''(\gamma) > 0$ for $\gamma \in [0, 1]$. Substantively, this means that defensive measures are not costly when none are taken ($\gamma = 1$), less defense is cheaper than more, and that the each defensive measure is more expensive to implement than the previous. Because $v_1 + v_2$ is not a function of $\gamma$ (unless $\gamma = 0$), we can rewrite it as $V^* > 0$. The first order condition is:

$$-V^* - g'(\gamma) = 0$$

If $g'(\gamma) = -V^*$ for some $\gamma \in [0, 1]$, the Target picks this value. It must be unique because $-g'(\gamma)$ is strictly decreasing, and it is a maximum because the second derivative is $-g''(\gamma)$, which is negative.

If $g'(\gamma) > -V^*$ for all $\gamma \in [0, 1]$, then the solution for the optimization problem lies beyond $\gamma = 1$, and the Target's utility is strictly increasing on the interval. It therefore chooses the maximum of that interval, $\gamma = 1$.

If $g'(\gamma) < -V^*$ for all $\gamma \in [0, 1]$, then the solution for the optimization problem lies before $\gamma = 0$, and the Target's utility is strictly decreasing on the interval. It therefore chooses the minimum of that interval, $\gamma = 0$. □

# 6 Works Cited

Arce M, Daniel G. and Todd Sandler. 2005. "Counterterrorism: A Game-Theoretic Analysis." *Journal of Conflict Resolution* 49 (2): 183-200.

Benmelech, Efraim, Claude Berrebi, and Esteban F. Klor. 2015. "Counter-Suicide-Terrorism: Evidence from House Demolitions." *Journal of Politics* 77 (1): 27-43.

Berry, Nicholas O. 1987. "Theories on the Efficacy of Terrorism." *Journal of Conflict Studies* 7 (1): 7-20.

Blomberg, S. Brock, Gregory D. Hess, and Akila Weerapana. 2004. "Economic Conditions and Terrorism." *European Journal of Political Economy* 20 (2): 463-478.

Bloom, Mia. 2005. *Dying to Kill: The Allure of Suicide Terror.* New York: Columbia University Press.

Brancati, Dawn. 2006. "Decentralization: Fueling the Fire or Dampening the Flames of Ethnic Conflict and Secessionism?" *International Organization* 60 (3): 651-685.

Bueno de Mesquita, Ethan. 2005a. "The Quality of Terror." *American Journal of Political Science* 49 (3): 515-530.

Bueno de Mesquita, Ethan. 2005b. "Conciliation, Counterterrorism, and Patterns of Terrorist Violence." *International Organization* 59 (1): 145-176.

Carter, David. 2016. "Provocation and the Strategy of Terrorist and Guerilla Attacks." *International Organization*, Forthcoming.

Clauset, Aaron, Lindsay Heger, Maxwell Young, and Kristian Skrede Gleditsch. 2010. "The Strategic Calculus of Terrorism: Substitution and Competition in the Israel-Palestine Conflict." *Conflict and Cooperation* 45 (1): 6-33.

Conrad, Justin and Kevin Greene. 2015. "Competition, Differentiation, and the Severity of Terrorist Attacks." *Journal of Politics* 77 (2): 546-561.

Crenshaw, Martha. 1981. "The Causes of Terrorism." *Comparative Politics* 13 (4): 379-399.

Cronin, Audrey Kurth. 2011. *How Terrorism Ends: Understanding the Decline and Demise of Terrorist Campaigns.* Princeton: Princeton University Press.

Dettke, Dieter. 2009. *Germany Says "No": The Iraq War and the Future of German Foreign and Security Policy.* Baltimore: Johns Hopkins University Press.

Dugan, Laura and Erica Chenoweth. 2012. "Moving Beyond Deterrence: The Effectiveness of Raising the Expected Utility of Abstaining from Terrorism in Israel." *American Sociological Review* 77 (4): 597-624.

Findley, Michael G. and Joseph K. Young. 2012. "More Combatant Groups, More Terror?: Empirical Tests of an Outbidding Logic." *Terrorism and Political Violence* 24 (5): 706-721.

Findley, Michael G. and Joseph K. Young. 2015. "Terrorism, Spoiling, and the Resolution of Civil Wars." *The Journal of Politics* 77 (4): 1115-1128.

Fortna, Virginia Page. 2015. "Do Terrorists Win? Rebels' Use of Terrorism and Civil War Outcomes." *International Organization*, Forthcoming.

Fromkin, David. 1975. "The Strategy of Terrorism." *Foreign Affairs* 53 (4): 683-698.

Hoyt, Timothy D. 2015. "Pakistan: A Tale of Two Allies." In *Coalition Challenges in Afghanistan: The Politics of Alliance* Eds. Gale Mattox and Stephen Grenier. Stanford: Stanford University Press.

Jaeger, David A., Esteban F. Klor, Sami H. Miaari, and M. Daniele Paserman. 2015. "Can Militants Use Violence to Win Public Support? Evidence from the Second Intifada." *Journal of Conflict Resolution* 59 (3): 528-549.

Kydd, Andrew H. and Barbara F. Walter. 2002. "Sabotaging the Peace: The Politics of Extremist Violence." *International Organization* 56 (2): 263-296.

Kydd, Andrew H. and Barbara F. Walter. 2006. "The Strategies of Terrorism." *International Security* 31 (1): 49-80.

Lake, David A. 2002. "Rational Extremism: Understanding Terrorism in the Twenty-First Century." *Dialogue IO* 1 (1): 15-29.

Laqueur, Walter. 1987. *The Age of Terrorism.* Boston: Little, Brown.

Lucas, Jr., Robert E. 1976. "Econometric Policy Evaluation: A Critique." In *The Phillips Curve and Labor Markets* Eds. Karl Brunner and Allan H. Meltzer. North

Holland: Elsevier.

Lum, Cynthia, Leslie W. Kennedy, and Alison Sherley. 2006. "Are Counter-Terrorism Strategies Effective? The Results of the Campbell Systematic Review on Counter-Terrorism Evaluation Research." *Journal of Experimental Criminology* 2 (4): 489-516.

Moghadam, Assaf. 2008. *The Globalization of Martyrdom: Al Qaeda, Salafi Jihad, and the Diffusion of Suicide Attacks.* Baltimore: Johns Hopkins University Press.

Nemeth, Stephen. 2014. "The Effect of Competition on Terrorist Group Operations." *Journal of Conflict Resolution* 58 (2): 336-362.

Netanyahu, Benjamin. 1995. *Fighting Terrorism: How Democracies Can Defeat Domestic and International Terrorists.* New York: Farrar Straus Giroux.

Pauly, Jr., Robert J. 2013. "French Security Agenda in the Post-9/11 World." In *Old Europe, New Europe, and the U.S.: Renegotiating Transatlantic Security in the Post 9/11 Era* Eds. Tom Lansford and Blagovest Tashev. Burlington: Ashgate.

Phillips, Brian J. 2015. "Deadlier in the U.S.? On Lone Wolves, Terrorist Groups, and Attack Lethality." *Terrorism and Political Violence*, Forthcoming.

Pillar, Paul R. 2001. *Terrorism and U.S. Foreign Policy.* Washington, DC: Brookings.

Powell, Robert. 2007. "Allocating Defensive Resources with Private Information about Vulnerability." *American Political Science Review* 101 (4): 799-809.

Price, Jr., H. Edward. 1977. "The Strategy and Tactics of Revolutionary Terrorism." *Comparative Studies in Society and History* 19 (1): 52-66.

Rashid, Ahmed. 2009. *Descent into Chaos: The U.S. and the Disaster in Pakistan, Afghanistan, and Central Asia.* New York: Penguin.

Rosendorff, B. Peter and Todd Sandler. 2004. "Too Much of a Good Thing: The Proactive Response Dilemma." *Journal of Conflict Resolution* 48 (5): 657-671.

Savun, Burcu and Brian J. Phillips. 2009. "Democracy, Foreign Policy, and Terrorism." *Journal of Conflict Resolution* 53 (6): 878-904.

Stanton, Jessica A. 2013. "Terrorism in the Context of Civil War." *Journal of Politics* 75 (4): 1009-1022.

Stedman, Stephen John. 1997. "Spoiler Problems in Peace Processes." *International Security* 22 (2): 5-53.

Walter, Barbara F. 2006. "Building Reputation: Why Governments Fight Some Separatists but not Others." *American Journal of Political Science* 50 (2): 313-330.

Wilkinson, Paul. 1993. "Politics, Diplomacy, and Peace Processes: Pathways Out of Terrorism." *Terrorism and Political Violence* 11 (4): 66-82.