1 Correcting the Mistake

For the situation in which only the unresolved type is willing to accept an offer of 0 (i.e., when $\alpha \in \left(\frac{Vp_B - c_B}{t_B}, \frac{V'p_B - c_B}{t_B}\right)$), state A makes the aggressive offer of $x = 0$ if:

$$q > \frac{c_A + \alpha t_A + \frac{c_B + \alpha t_B}{V'}}{p_B + c_A + \alpha t_A}$$

Note that this is the corrected version from class. (The original denominator I presented was incorrect.)

Because A makes the aggressive offer if q is greater than that amount, increasing the right hand side of the inequality makes it harder for the parameters to fall within that region, which thereby increases the chances that A makes the aggressive offer that generates a positive probability of war. Thus, increasing α by ϵ decreases the overall probability of war if:

$$\frac{c_A + (\alpha + \epsilon)t_A + \frac{c_B + (\alpha+\epsilon)t_B}{V'}}{p_B + c_A + (\alpha + \epsilon)t_A} > \frac{c_A + \alpha t_A + \frac{c_B + \alpha t_B}{V'}}{p_B + c_A + \alpha t_A}$$

Substantial algebraic manipulation generates the following:

$$\frac{(p_B + c_A)t_B}{V'(p_B + c_A + \alpha t_A)(p_B + c_A + (\alpha + \epsilon)t_A)} > \frac{(c_B - V'p_B)t_A}{V'(p_B + c_A + \alpha t_A)(p_B + c_A + (\alpha + \epsilon)t_A)}$$

This corrects for the mistake in class.[1]

As was noted in class, the denominators on both sides is identical and positive. We may therefore delete them, leaving us with:

$$(p_B + c_A)t_B > (c_B - V'p_B)t_A$$

Dividing both sides by the positive trade values and multiplying each side by -1 (remembering to flip the inequality), we can rework that as:

$$\frac{V'p_B - c_B}{t_B} > -\frac{p_B + c_A}{t_A}$$

Note that the right side of the inequality must be negative. Meanwhile, the left side must be positive. This is because the parameters we are focusing on here have $\alpha \in \left(\frac{Vp_B - c_B}{t_B}, \frac{V'p_B - c_B}{t_B}\right)$. The second part of this constraint implied that the resolved type was unwilling to accept an offer of 0, or $p_B - \frac{c_B + \alpha t_B}{V'} > 0$. This can be written as $\frac{V'p_B - c_B}{t_B} > \alpha$. Since α is positive and $\frac{V'p_B - c_B}{t_B}$ is greater than it, $\frac{V'p_B - c_B}{t_B}$ must be positive as well.

What this all means is that the inequality in question has a positive left side and a negative right side. Consequently, the inequality must hold. In turn, increasing the value of trade (by increasing α), decrease the parameters under which war occurs.

2 A Better Substantive Motivation

A question early on in class today asked about measuring the costs of war and the value of trade as a means of applying the model to a historical example (or many historical examples). My initial

[1] Specifically, I had listed the numerator of the right hand side as $(V'p_B - c_B)t_A$, flipping the negative within the parenthese. Unfortunately, that negative is pivotal for proving that the inequality holds. I copied this down wrong before leaving for class this morning, causing this part of the lecture to explode. Oops.

response was that while data are readily available on trade, there really aren't good ways to measure the costs of war. But we still care about how manipulating the costs of war affects the outbreak of war, and modeling helps us get at that in the absence of empirical data. That answer may have been unsatisfying, so I want to expand the thought in two ways and clarify what I meant.

First, I should have emphasized that *we as analysts* do not have good ways to measure the costs of war across temporal and global domains. Every year, the CIA and Department of Defense each spend *billions* to answer those questions. And that should not be surprising—the costs of war incorporate how much a government cares about the circumstances, what the various possible ways a war could play out, the probability each of those outcomes occurs, the costs of each of those outcomes, and so forth. It would be ridiculous for a social scientist—even one armed with the largest research budget in the history of academia—to think he or she could get close to the exact potential costs of war for even a single potential conflict, never mind all potential conflicts across time.[2] And we don't even have access to classified information, which is integral to getting the answers right!

The key takeaway here is not that costs of war and values of trade are impossible to calculate. On the contrary, as you are reading this, a sizeable chunk of the United States' bureaucracy is hard at work trying to answer those exact questions. And we do a good job of it. Rather, the point is this task is not in the financial domain of social science, so researchers have to focus their priorities elsewhere.

Second, there is a deeper theoretical question of how changing inputs changes outputs. Thoerists dating all the way back to Immanual Kant (1795) have argued that increasing trade should reduce international hostilities, ostensibly because it increases the cost of war. And this has become an integral part of U.S. foreign policy and the international community's push for peace since the end of World War II, with the formations of the GATT and World Bank. Policy makers may therefore wish to know whether trade actually has this pacifying affect. That answer should be general—we wouldn't want to know whether the effect holds when $p_A = .55$, $c_A = .1$, and $c_B = .12$ because that is only a single dot in the parameter space and we are applying the policy *everywhere*.

This is where models are invaluable. If we have a functional model of international negotiations, we can answer this question by conducting comparative statics because the exact values of the parameters are intentionally general.[3] Thus, even if we don't know the exact values of p_A, c_A, or c_B, we can still say sensible things about whether trade will decrease the probability of war.

3 A Brief Note of Regret

I thought we would get through this model a bit faster today. The key takeaway was supposed to be that the probability of war can increase as trade grows, not decrease as one might guess.[4] But we never got there!

At the beginning of this semester, I mentioned that this is my first time teaching this particular class, though I have taught these models in different forms in previous classes. So please bear with me as I figure out how much we can accomplish during a single class period.[5] Apologies, and thanks!

[2] Depending on how you count, there are about 200 countries in the world. If we *only* focus on pairs of countries fighting (and not think about alliance dynamics), you have 19,900 possible wars in each year. With two costs to calculate per war, you are up to 39,800. But you'd want to do this over many years of data. Let's call it 50 years. Suppose by some miracle you could calculate each of these costs in an hour. It would still take 227 *years* to complete the dataset.

[3] A similar logic guides statistical social science. For example, quantitative scholars of war do not try to calculate specific cost values of conflict. Rather, they find data that measure whether costs are comparatively higher in one case versus another (e.g., does either side have nuclear weapons?) and see whether whether war is more or less likely under those circumstances.

[4] As we showed today, this is a short-run effect. The probability of war eventually decreases once costs of war grow large enough.

[5] I think next semester I will begin with the interior solution (which we will cover at the beginning of lecture on Thursday) and just briefly explain why it's a short-run effect rather than formally solve for them as we did in class today.