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Reminder: Unless otherwise specified, you may assume the receiver accepts when indifferent and only solve for interior (non-corner) solutions throughout this problem set. You may also assume that a player's payoff for war is positive (i.e., $p - c > 0$).

1 Multiple Outcomes

In the standard complete information setup, all wars ended in complete victory or complete defeat. Suppose instead that A wins and takes the entire good with probability p_A, B wins and takes the entire good with probability p_B, and a stalemate occurs with probability $1 - p_A - p_B$. In the event of a stalemate, A receives .6 of the good and B receives .4 of it. Regardless of the outcome, states always pay their war costs as normal.

a) Write each side's expected value for war.

b) Prove that a mutually preferable peaceful alternative x always exists. You may find it illuminating to illustrate the problem geometrically, but remember that pictures are not proofs.

2 Mutual Uncertainty

A common misconception is that uncertainty over the likely outcome of conflict will lead to bargaining breakdown. This is not true when both sides face the same uncertainty. Imagine a scenario where the likelihood of victory depends heavily on military cohesion. Yet there is no way to know how cohesive the troops are without actually fighting. Even so, both states believe that B's troops will be cohesive with probability q and uncohesive with probability $1 - q$. In the first case, the A wins with probability p_A; in the second, it wins with probability p'_A. (Essentially, cohesion is good for the B's probability of victory.) Regardless of the cohesion, the actors pay the same costs as before. Note that both sides know exactly what the other knows and nothing more.

a) Write each side's expected value for war. (Hint: It may help to draw a diagram that maps out the probability of each outcome occurring.)

b) Prove that, despite the uncertainty, a mutually preferable peaceful alternative x always exists.

c) In contrast to this problem, we know that *asymmetric* uncertainty about the probability of victory can lead to war. Briefly explain why asymmetric uncertainty creates bargaining problems but symmetric uncertainty does not.

d) Suppose that both sides knew that A wins with probability p_A but believed that B's costs are c'_B with probability q and c_B with probability $1 - q$. (Again, both sides are equally informed of these probabilities.) Redo (a) and (b) for this setup.

3 Risk Aversion

In the standard setup, we assumed that individuals had *risk neutral* preferences. Thus, given a value of 1 with probability p and a value of 0 with probability $1-p$, an offer of p made an individual indifferent between taking that amount with certainty and receiving the random outcome.

Some of the older literatures on conflict argued that states are *risk averse*; that is, they would be willing to take a little bit less than the expected outcome for war if they could secure that amount with certainty.

Another way to think about this is that states have decreasing marginal returns for each additional amount they receive. So the first unit of the good provides a larger value to the state than the second unit of the good, the second unit provides a larger value to the state than the third unit, and so forth.

The square root function models this type of preference. For example, suppose state A's payoff for it share of the bargaining good is $\sqrt{x}$. Receiving none of the good therefore gives a payoff of 0. Receiving .1 of the good increases that payoff to ≈ 0.316. Receiving .2 of the good further increases that payoff to $\approx .447$. But note that the marginal increase in utility between 0 and .1 was larger than the marginal increase in utility between .1 and .2.

Suppose war is free (i.e., $c_A, c_B = 0$) and states value a share of a bargain at the square root of their share. Does a range of outcomes mutually preferable to war exist?

4 Nuclear Deterrence

Suppose State A has a small nuclear arsenal. Some military strategists doubt that nuclear weapons have any tactical use. Let's take this seriously and imagine that these nuclear weapons have no effect on the probability of victory in war, so that p_A remains p_A.

Instead, imagine that State A might use them in the event of being militarily defeated. Normally, we assume that war's winner takes everything. Now suppose that State B chooses an amount y to give to A if B wins the war. B keeps the remainder, $1 - y$. Unlike before, State A cannot accept or reject this—State A assuredly claims y and State B assuredly receives the remainder $1 - y$.

Nevertheless, State A might wish to use nuclear weapons. Imagine that, after receiving the value y, State A chooses whether to nuke its opponent. Its payoff for not nuking in this phase is simply y; its payoff for nuking is $y + \sqrt{1-y} - \alpha$, where $\sqrt{1-y}$ reflects a "vengeance" payoff and $\alpha \in (0, 1)$ reflects the costs of breaking international norms against nuclear non-use. Note that the vengeance payoff is decreasing in y, meaning that State A finds nuking its enemy less attractive as it receives a larger share of the post-war settlement. If B is nuked, B suffers a cost $n > 1$.

a) Imagine that A has lost the war. For what values of y will A nuke B?

b) Suppose that A does not nuke when indifferent. What is B's optimal claim y?

c) Imbed this interaction into a standard crisis bargaining model. What is the range of settlements mutually preferable to war?

d) Nuclear skeptics argue that countries should dismantle their atomic arsenals because they have no tactical use. Compare the bargaining range in (c) to the range in the standard model. Use that comparison to comment on the nuclear skeptics' argument.

5 Private Benefits

The standard setup assumes that states are unitary actors. Here, suppose the A is a unitary actor but B has a leader who controls her group's decision to go to war. The leader still internalizes the 1 unit of value if B wins and the cost c_B as before. However, if B wins, she derives some private benefit $b > 0$. (This could be because it will boost her ego, give her a longer page on Wikipedia, or ensure a lifetime of steak dinners.) Despite the leader's bias for war, the purpose of this question is to show that peaceful agreements can still work provided that b is not too large.

a) Write A's and B's leader's expected values for war.

b) What is the maximum value of b such that a peaceful settlement still exists? Hint: Attempt to prove the existence of a peaceful settlement as normal. Then rewrite the final inequality in terms of b.

c) Does the likelihood that a settlement exists increase or decrease in p_B. Why?

d) Multiply the inequality in part (b) by p_B and interpret its meaning substantively. That is, explain what it represents in English without using any mathematical symbols.

6 Shrinking Pie Costs

Rather than the states paying a direct cost of war, suppose instead that war reduces the overall size of the bargaining good to $\pi \in (0, 1)$.

a) Solve the preventive war model using this modification.

b) Suppose that B is privately informed whether the reduction in costs is $\pi' > \pi$ or π. Specifically, A believes the pie shrinks to π with probability q and shrinks to π' with probability $1 - q$. Solve this incomplete information game.

c) Does your answer to part (b) seem more similar to uncertainty over B's costs of war (when war values are not interdependent) or uncertainty over the probability of victory (when war values are interdependent)?

7 Uncertainty over the Value of War

Consider the standard model with the following twist: Nature begins the game by drawing the value of the prize as $\pi \in (0, 1)$ with probability q and 1 with probability $1 - q$. State B observes the draw but State A does not. One could conceptualize this as State B knowing the quantity of natural resources in a disputed territory or the sum value of tax revenues of people living in the area. The game proceeds as normal: State A makes a take-it-or-leave-it offer, and State B accepts or rejects.

a) Solve this incomplete information game.

b) With uncertainty over the probability of victory, the type that sometimes fights in equilibrium is the stronger of the two. With uncertainty over the costs of war, the type that sometimes fights in equilibrium has the lower costs of the two. In other words, the "better" type is the one that initiates war. Is that also true here? Explain the logic for why or why not.

8 Monotonicity and Uncertainty

Consider the standard two-type incomplete information ultimatum crisis bargaining model in which the receiver has costs c'_B with probability q and has costs $c_B < c'_B$ with probability $1 - q$. A common argument that comes from the bargaining model of war literature is that decreasing uncertainty decreases the probability of war. This question asks you to use comparative static analysis to investigate whether that is true.

a) Note that the game converges to the complete information case as q goes to 0 or 1. Thus, we could think of $q = .5$ as maximizing the proposer's uncertainty. Does moving the prior belief q away from .5 (in either direction) monotonically decrease the probability of war?

b) Another way to conceptualize uncertainty is the size of $c'_B - c_B$. Note that when $c'_B - c_B = 0$, the game converges to the complete information case. Thus, as $c'_B - c_B$ decreases, so does the uncertainty. Does decreasing $c'_B - c_B$ monotonically decrease the probability of war? (Hint: Compare the equilibrium outcome of the game when the possible costs are c'_B and c_B to the equilibrium outcome of the game when the possible costs are $c'_B - \epsilon$ and $c_B + \epsilon$.)

c) Repeat part (a) for the standard two-type incomplete information ultimatum crisis bargaining model where costs are complete information but B has private information about the probability of victory—explicitly, A's probability of victory is p_A with probability q and its probability of victory is $p'_A > p_A$ with probability $1 - q$.

d) Repeat part (b) for incomplete information game described in part (c). (Hint: Compare the equilibrium outcome of the game when the possible probabilities of victory are p'_A and p_A to the equilibrium outcome of the game when the possible costs are $p'_A - \epsilon$ and $p_A + \epsilon$.)

9 The Price of Uncertainty

The "price of uncertainty" is an uninformed player's utility loss when he transitions from a game of complete information to a game of incomplete information. (See this video for more information and an example.) We will calculate this for the standard two-type incomplete information ultimatum crisis bargaining model in which the receiver has costs c'_B with probability q and has costs $c_B < c'_B$ with probability $1 - q$.

a) What is state A's expected utility for this incomplete information game? Your answer may take on two different functional forms depending on q.

b) Suppose state A knew that B's cost was c'_B. What is B's expected utility for the game? Multiply this value by q. Now add that to B's expected value for the game where A knows B's cost is c_B multiplied by $1 - q$. This is A's expected utility for the complete information analogue of the incomplete information game, where Nature informs both players of B's costs.

c) The price of uncertainty is your answer from (b) minus your answer from (a). Write this out. Again, your answer may take on two different functional forms depending on q.

d) Suppose A could pay an intelligence organization to uncover B's private information. Naturally, when the price of uncertainty is high, A would be willing to pay more. Using your answer to (c), for what values of q is A more willing to pay for that intelligence organization? For what values of c_A is A more willing to pay for it?

10 Is Commitment to Peace Credible after Screening?

In incomplete information environments, accepting an offer reveals private information to the opponent—namely, that the offer was preferable to war. This might raise suspicion that the proposer will update its belief that the opponent was weaker than initially expected. Given that agreements must be self-enforcing to be sustainable, one may then wonder whether the proposer would rescind the original agreement and try to offer something even smaller instead.

To investigate this, consider the following setup with incomplete information over costs and three types. Suppose Nature draws B's costs of war as $c_B = .1$ with probability .3, $c_B = .25$ with probability .4, and $c_B = .35$ with probability .3. B knows its cost but A only knows this prior belief. In negotiations, A makes an ultimatum offer, which B accepts or rejects. Accepting implements the settlement; rejecting leads to war payoffs. Suppose in this case that A wins with probability $p_A = .5$ and A's costs are $c_A = .2$.

a) Find A's optimal offer for this standard setup.

b) Suppose the states play the game as before. If the states reach a peaceful settlement, A knows B must not be a type that would have fought. Imagine that A has the opportunity to rescind its original offer to the types that accepted. Would it want to? (Hint: Imagine this were a new game that excluded any types that rejected. Is the equilibrium offer of this new game identical to the offer accepted in the original game? Another hint: Be careful to properly assign prior beliefs in the revised game, recalling that the probability distribution must sum to 1.)

c) Substantively, why does the proposer rescind or not rescind the offer?

11 Cheap Talk

Consider the standard incomplete information game in which A is uncertain whether B's cost is c'_B or c_B. Imagine that B is allowed to send a cheap talk message to A before the start of the game. Is truth telling an equilibrium? (That is, is it an equilibrium for the high cost type to report that it is a high cost type and for the low cost type to report that it is a low cost type?)

12 Uncertainty about the Proposer's Cost

In the game with uncertainty over costs, A does not know B's cost for war. Imagine instead that B is uncertain over A's cost instead. Specifically, with probability q, A's cost of war is $c'_A > c_A$ and with probability $1 - q$ it is c_A. A knows its own cost but B only knows this prior distribution.

Find the equilibrium of the game with the standard bargaining protocol—i.e., A makes an ultimatum offer $x \in [0, 1]$, which B accepts or rejects.