Contents

Reminder: Unless otherwise specified, you may assume the receiver accepts when indifferent and only solve for interior (non-corner) solutions throughout this problem set.

1 Bargaining over Weapons

Consider the following game within the bargaining model of war framework. Two actors, D and R, bargain over a good valued 1 over two periods, with the second period's payoff weighed by $\delta > 0$. D begins the game by offering $x \in [0, 1]$. R has three options: accept, reject, or build. Accepting ends the game with that division. R earns $(1 + \delta)x$ and D earns $(1 + \delta)(1 - x)$. Rejecting leads to war. R wins with probability p_R and both states pay costs. These payoffs carry over to the next period. Thus, R earns $(1 + \delta)(p_R - c_R)$ and D earns $(1 + \delta)(1 - p_R - c_D)$.

If R builds, it pays a cost $k > 0$ to do so. D then chooses whether to launch preventive war or not. Each player's payoffs for preventive war are identical to the rejection payoff, except R also subtracts out k. If D chooses not to, takes back the entire good for the period, and the states advance into the next period. (In this manner, the deal is a *quid-pro-quo* continent on R accepting the offer and not building.)

In the second stage, D offers $y \in [0, 1]$ and R accepts or rejects. Accepting yields an overall payoff of $\delta y - k$ for R and an overall payoff of $1 + \delta(1 - y)$ for D. Rejecting leads to war payoffs. However, R now prevails with probability $p'_R > p_R$ because of its new weapons. Thus, the overall payoffs are $\delta(p'_R - c_R) - k$ for R and $1 + \delta(1 - p'_R - c_D)$ for D.

a) Show that the equilibrium offer D makes in the second stage is $y = p'_R - c_R$. (Hint: Is the second stage any different from what we covered in class?)

b) Show that D prevents if $p'_R > \frac{p_R + c_D}{\delta} + p_R + c_D + c_R$.

c) Suppose $p'_R > \frac{p_R + c_D}{\delta} + p_R + c_D + c_R$. Prove that D offers $x = p_R - c_R$ and R accepts that in equilibrium.

d) Suppose $p'_R < p_R + \frac{p_R - c_R + k}{\delta}$. Prove that D offers $x = p_R - c_R$ and R accepts that in equilibrium.

e) Suppose that $p'_R \in \left(p_R + \frac{p_R - c_R + k}{\delta}, \frac{p_R + c_D}{\delta} + p_R + c_D + c_R\right)$. Prove that D offers $x = \frac{\delta(p'_R - c_R) - k}{1 + \delta}$ and R accepts that in equilibrium.

Note that this covers the entire parameter space. Thus, by showing that R accepts an offer in each case, you have shown that no weapons are ever built in equilibrium.

2 Grim Trigger Practice

Consider the following game:

	Left	Right
Up	$x, 6$	$y, 9$
Down	$4, 0$	$2, 3$

Players play this game over an infinite horizon with common discount factor $\delta \in (0, 1)$, where $4 > x > 2 > y > 0$.

a) Let $y = 1$. Consider the following trigger strategies: player 1 plays up and player 2 plays left in every period; if any player ever deviates from that strategy, player 1 plays down and player 2 plays right. For what values of δ is this an equilibrium? How does this change as x increases? Would you recommend that an international institution increase x as a means of incentivizing cooperative behavior?

b) Let $x = 3$. For what values of δ are those trigger strategies an equilibrium? How does this change as y increases? Would you recommend that an international institution increase y as a means of incentivizing cooperative behavior?

3 Issue Linkage

One way international institutions can promote cooperation is by linking multiple issues together. This question verifies that intuition using a repeated prisoner's dilemma.

a) Consider the prisoner's dilemma below:

	Left	Right
Up	$3, 2$	$-2, 4$
Down	$4, -2$	$0, 0$

What is the minimum value of δ necessary for the players to sustain mutual cooperation? (To answer this question, find the minimum value of δ necessary for player 1 to be willing to play a mutual grim trigger subgame perfect equilibrium. Then find the minimum value of δ necessary for player 2 to be willing to do the same. The answer is the *maximum* of these two values because both inequalities must be fulfilled for both players to be willing to play the strategy.)

b) Consider the prisoner's dilemma below:

	Left	Right
Up	$2, 3$	$-2, 4$
Down	$4, -2$	$0, 0$

What is the minimum value of δ necessary for the players to sustain mutual cooperation in this game?

c) Consider the prisoner's dilemma below:

	Left	Right
Up	$5, 5$	$-4, 8$
Down	$8, -4$	$0, 0$

Once more, what is the minimum value of δ necessary for the players to sustain mutual cooperation?

d) Note that the game in part c is the summation of the games from parts a and b. In essence, the game ties cooperation decisions on the issue from part a together with cooperation decisions on the issue from part b. Suppose $\delta = \frac{2}{5}$. Are the players better off keeping the issues separate or tying them together?

e) Suppose the institution is costly to create—i.e., bundling the issues together costs each state c to pay for coordination bureaucracy. If $\delta = \frac{2}{5}$, what is the maximum value of c the states would be willing to pay to create the institution?

4 Institutions as Interaction Accelerators

Another argument for how institutions encourage cooperation is that they decrease the amount of time between interactions, allowing states to more quickly trigger their grim triggers. This question explores that intuition.

a) Consider the following game:

	Left	Right
Up	$3, 3$	$1, 6$
Down	$6, 1$	$2, 2$

For what values of δ is both states playing grim trigger a subgame perfect equilibrium?

b) Imagine that institutions allows the players to change their strategy twice as fast as they could before. Thus, in each period, the states receive half as much as did previously, but they reap those payoffs twice as often. Thus, the new payoff matrix looks like this:

	Left	Right
Up	$\frac{3}{2}, \frac{3}{2}$	$\frac{1}{2}, 3$
Down	$3, \frac{1}{2}$	$1, 1$

Increasing the speed of the interaction in this manner requires manipulating how payoffs accumulate. To illustrate why this is the case, note that maintaining the same discount factor as before gives a player $\frac{\frac{3}{2}}{1-\delta}$ for mutual cooperation forever, which is strictly less than the $\frac{3}{1-\delta}$ that player received before the institution altered the interaction. This does not make sense, as the player should still accumulate the same payoff for the same set of strategies.

We can fix this by using $\delta' = \frac{1+\delta}{2}$ as the discount factor for this altered game. For what values of δ is both states playing grim trigger a subgame perfect equilibrium? (Hint: You should setup the inequalities in terms of δ' but ultimately solve for δ.)

c) How do your answers to (a) and (b) compare? Does the institution succeed in promoting co-operation?

5 Perverse Incentives

Consider the standard two type crisis bargaining game with uncertainty about the probability of victory with the following minor modification: in the event of bargaining breakdown, an international institution exerts effort to mitigate some of the costs. There are many ways to conceptualize this, but one frequent example is the United Nations High Commissioner for Refugees providing humanitarian aid to those displaced by war. To formalize this, imagine that with the international institution, the respective states only pay $\alpha_A c_A$ and $\alpha_B c_B$ if they fight, where $\alpha_A, \alpha_B \in (0, 1)$.

States are not naive and know that the institution will act in the event of bargaining breakdown. Solve the game when the states anticipate that their true expected costs to be only $\alpha_A c_A$ and $\alpha_B c_B$. How does this compare to the case when the institution does not exist (i.e., when $\alpha_A = \alpha_B = 1$). Explain why this result is different from what we saw in class when there was uncertainty of the cost of bargaining breakdown.

6 Moral Hazard

Suppose the United Nations wishes to hire a village leader to stop criminals from robbing raiding U.N. conveys as they pass through the area. The U.N. values safe passage at V. The village leader does not care one way or the other whether the U.N.'s convey is robbed. However, exerting effort to stop the criminals costs the leader $c > 0$, where $V > c$.

The United Nations faces an observation problem. Specifically, they cannot tell whether the village leader actually exerts effort to stop the criminals. All they know is whether the convoy was robbed at the end of the day. Suppose that the leader will assuredly stop the criminals if she exerts effort, but that the convoy will only pass through safely with probability $p \in [0, 1)$ if she does not exert effort; this might be because the criminals were waiting at wrong time or were setting up a trap on the wrong road.

a) Analyze the strategic constraints of this interaction in the following way. The U.N. begins by offering an amount x. After seeing the offer, the leader chooses whether to exert effort or not. The U.N. pays out x if and only if the convoy makes it through safely. (It cannot observe whether the leader exerted effort and therefore cannot pay out based on that.) For what values of V does the U.N. make an offer strictly greater than 0? When the U.N. makes such an offer, what is the offer size it picks? You may assume the leader exerts effort when indifferent.

b) When an offer strictly greater than 0 is made, how does that offer size change as p increases. Explain the intution.

c) Now suppose that the U.N. could observe the leader's effort decision and paid x based on that. For what values of V does the U.N. make an offer strictly greater than 0? When the U.N. makes such an offer, what is the offer size it picks?

d) How do the efficiency of the games in (a) and (c) compare? What does this say about the value of observable actions in international relations?

7 Gatekeeping versus Veto Power

An executive and legislature have respective ideal points $x_E, x_L \in \mathbb{R}$ for a policy with status quo $q \in \mathbb{R}$. They are negotiating to change that status quo. Their payoffs for a policy outcome x are the negative Euclidean distance between the actor's ideal point and that outcome.

a) **Gatekeeping.** Suppose the law gives the executive gatekeeping power. That is, the executive first chooses whether to permit the legislature to change the policy. If he does not, the status quo persists. If he does, the legislature implements a policy $x \in \mathbb{R}$. Find the SPE.

b) **Veto Power.** Suppose the law allows the executive to veto unacceptable policies. That is, the legislature first proposes a policy $x \in \mathbb{R}$. The executive sees that proposal and accepts or rejects it. Accepting implements that policy; rejecting maintains the status quo. Find the SPE.

c) Compare welfare between the two legislative rules.

8 Mechanism Design Practice

Consider an incomplete information game between State A and State B. Both have private information about their capabilities. State A is strong with probability $\frac{1}{2}$ and weak with probability $\frac{1}{2}$; likewise, State A is strong with probability $\frac{1}{2}$ and weak with probability $\frac{1}{2}$. (These probabilities are independent of each other.) Each state knows which it is, but the other only has the prior belief. If both are strong or both are weak, the probability A wins a war is .5. If A is strong and B is weak, A wins with probability .7. If A is weak and B is strong, A wins with probability .3. War costs both parties .05.

a) Consider a direct mechanism that instructs all types to fight regardless of the type that they report. Is this direct mechanism individually rational? Is it incentive compatible? Is it efficient?

b) Consider a direct mechanism that gives all types for each state a value of .5 regardless of the type that they report. Is this direct mechanism individually rational? Is it incentive compatible? Is it efficient?

c) Consider a direct mechanism that gives each party .5 if they both report that they are strong or both report that they are weak and gives .7 to the state that reports strong and .3 to the state that reports it is weak if they report opposite types. Is this direct mechanism individually rational? Is it incentive compatible? Is it efficient?

9 Existence of Efficient Mechanisms

Suppose Nature allots State 1 with 1, 2, or 3 military units, each with probability $\frac{1}{3}$. Call that draw m_1. Nature also allots State 2 with 2, 3, or 4 military units, each again with probability $\frac{1}{3}$. Call that amount m_2. Each draw is independent, and each state observes its own draw but only knows the prior distribution for the other state.

If the states fight a war, State 1 wins with probability $\frac{m_1}{m_1+m_2}$. Note that this probability is increasing in State 1's military allotment and decreasing in State 2's. State 2 wins with complementary probability. For simplicity, suppose that the costs $c_1 > 0$ and $c_2 > 0$ are not a function of the military allotments.

Consider the standard mechanism design framework for the bargaining model of war. Specifically, the mechanism asks for each state to report its type. Based on the types reported, the mechanism then assigns an efficient settlement or war as the outcome.[1] For what values of $c_1 + c_2$ do efficient, incentive compatible, and individually rational mechanism not exist?

[1] It may assign these probabilistically, of course.