Basic Models of Conflict and Cooperation

Due Beginning of Class February 6, 2019

Group Work Encouraged, But Write Up Must Be One's Own

No Late Work Accepted

In class, we saw learned two basic games that model competition and cooperation between states: the prisoner's dilemma and the stag hunt. This problem set will test your knowledge of these strategic interactions. For all questions other than the fifth, assume that the actors only play the game once.

1) Consider the following game:

	Left	Right
Up	$9, 7$	$1, 2$
Down	$5, -1$	$2, 0$

a) For what opposing strategies is player 1 willing to play up?

b) For what opposing strategies is player 2 willing to play left?

c) Is this a prisoner's dilemma or a stag hunt? (Remember, you should focus more on the *ordering of an individual's preferences* than the actual payoff values.) Is it possible for the players to obtain an efficient outcome?

2) Consider the following game:

	Left	Right
Up	$3, 1$	$0, 6$
Down	$7, -6$	$1, 0$

a) For what opposing strategies is player 1 willing to play up?

b) For what opposing strategies is player 2 willing to play left?

c) Is this a prisoner's dilemma or a stag hunt? Is it possible for the players to obtain an efficient outcome?

3) Consider the following game:

	Left	Right
Up	$7, 0$	$3, 1$
Down	$6, -4$	$4, -1$

The above game is neither a prisoner's dilemma nor a stag hunt. Nevertheless, you can use what you have learned to make a good prediction about the outcome of the game.

a) Under what conditions would player 1 want to play up? Under what conditions would player 1 want to play down? Based on this information alone, can we conclude what player 1 should do?

b) Under what conditions would player 2 want to play left? Under what conditions would player 2 want to play right? Based on this information alone, can we conclude what player 2 should do?

c) An important part of strategic reasoning is putting yourself into the other player's shoes, thinking about how they will play, and then choosing your strategy based on that information. With that in mind, what outcome should we expect from this game? Why? Is this outcome efficient?

4) In class, we discussed how first strike advantages lead to preemptive war. This question will relate that to the basic models of conflict and cooperation.

Players 1 and 2 are in conflict over a strip of territory valued at 1. They first must decide whether to *bargain* or *fight*. If they both bargain, suppose that player 1 will ultimately keep .55 of the territory and player 2 will keep .45. If they both fight, player 1 expects to win the war with probability .6 and player 2 expects to win the war with probability .4, but both will pay .05 in costs. If one bargains while the other fights, the fighter receives a first strike advantage: he will be .2 more likely to win the war, which also makes the other party .2 *less* likely to win. (The base probabilities of victory are the same as before. If any player starts a war, *both* states pay their costs.)

a) Draw a 2x2 game matrix. Label the strategies as *bargain* and *fight*. Calculate and fill in each player's payoff for the corresponding outcomes.

b) Is the game a prisoner's dilemma or stag hunt?

5) Recall the prisoner's dilemma using the payoffs from lecture:

	Left	Right
Up	$1, 1$	$-1, 2$
Down	$2, -1$	$0, 0$

In an infinitely repeated version of this, we saw that the players could maintain cooperation if the probability that the interaction would continue (usually called the *discount factor*) was greater than $\frac{1}{2}$.

Now consider this slightly modified version, in which the "sucker's" payoff is worse:

	Left	Right
Up	$1, 1$	$-5, 2$
Down	$2, -5$	$0, 0$

a) Suppose the players cooperate for all of time. Let p be the probability the interaction continues after every round. What is each player's total payoff?

b) Using grim trigger strategies, what is the minimum value of p for which the players are willing to maintain mutual cooperation?

c) Generalizing from this, what can we say about how changing the sucker's payoff affects the possibility of cooperative behavior between states? In words, explain why this is the case.

d) Now suppose the players' payoffs looked like this:

	Left	Right
Up	$1, 1$	$-1, 6$
Down	$6, -1$	$0, 0$

Note that this is the same game as discussed in class, except the temptation payoff is greater for both players. Suppose the players cooperate for all of time. Let p be the probability the interaction continues after every round. What is each player's total payoff?

e) Using grim trigger strategies, what is the minimum value of p for which the players are willing to maintain mutual cooperation?

f) Generalizing from this, what can we say about how changing the temptation's payoff affects the possibility of cooperative behavior between states?